Gravitational Wave Signature of a Mini Creation Event

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Introduction

- Ten black hole binary coalescences confirmed detections + one neutron star - neutron star coalesence.
- Other possible astrophysical sources - continuous wave, stochastic, bursts, etc.
- We propose another astrophysical source - the mini creation event (MCE) and investigate its detectability.

Quasi Steady State Cosmology

The expansion of the universe is described by the scale factor:

$$S(t) = \exp(t/P)[1 + \xi \cos(2\pi t/Q)]$$

Typically, we may take $P \approx 10^3$ Gyr and $Q \approx 50$ Gyr and $\xi = 0.8$

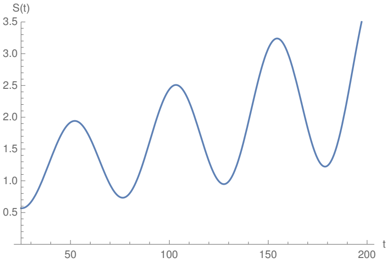

Figure: The figure shows the scale factor as a function of time.

Close to each minimum, matter is created by MCEs so as to retain the same density (Steady State) in each cycle.

MCE model - Bianch Type I

MCE model:

Approximated by a triaxial ellipsoid expanding anisotropically from some minimum size

Source frame: (x, y, z)

Principal axes of MCE chosen as (x, y, z)

Expansion approximately described by Bianchi Type I model metric:

$$ds^2 = c^2 dt^2 - X^2(t)\ dx^2 - Y^2(t)\ dy^2 - Z^2(t)\ dz^2$$

Quadrupole Tensor and Formula

Quadrupole tensor:

$$I_{xx} = \frac{1}{5}MX^2(t),\ \ I_{yy} = \frac{1}{5}MY^2(t),\ \ I_{zz} = \frac{1}{5}MZ^2(t)$$

where,

$$X(t) = (GM)^{1/3} t^{2/3} \left(\frac{2}{9}\right)^{\frac{2}{3}\sin\gamma - \frac{1}{3}}$$

and $Y(t), Z(t)$ described by $\gamma \longrightarrow \gamma + 2\pi/3,\ \ \gamma + 4\pi/3$.

GW amplitude in the TT gauge - quadrupole formula:

$$h_{ik}^{TT}(R,t) = \frac{2G}{c^4}\frac{1}{R}\left[\ddot{I}_{ik}(t - R/c)\right]^{TT}$$

GW Amplitudes in the Wave Frame

Source frame: (x, y, z) Wave frame: (X, Y, Z)

$(x, y, z) \longrightarrow (X, Y, Z)$ Euler angles (Goldstein convention): (α, ι, β)

Define basic GW amplitudes:

$$\begin{aligned} \mathfrak{h}_k(\gamma) &= \frac{\mathcal{A}}{R}\left(t - \frac{R}{c}\right)^{-2/3}\left(\frac{2}{9}\right)^{\frac{4}{3}\sin\left(\gamma+(k-1)\frac{2\pi}{3}\right)}, \quad k = 1, 2, 3 \\ \mathcal{A} &= \frac{4}{5}\left(\frac{2}{9}\right)^{1/3}\frac{(GM)^{5/3}}{c^4} \end{aligned}$$

and

$$h_1(\gamma) = \frac{1}{2}(\mathfrak{h}_1 - \mathfrak{h}_2) \qquad h_2(\gamma) = \frac{1}{4}(\mathfrak{h}_1 + \mathfrak{h}_2 - 2\mathfrak{h}_3)$$

GW amplitudes in Wave Frame:

$$\begin{aligned} h_+ &= \left[\frac{1}{2}(1+\cos^2\iota)\cos 2\alpha\cos 2\beta - \cos\iota\sin 2\alpha\sin 2\beta\right]h_1(\gamma) \\ &+ \sin^2\iota\ \cos 2\beta\ h_2(\gamma) \\ h_\times &= \left[\frac{1}{2}(1+\cos^2\iota)\cos 2\alpha\sin 2\beta + \cos\iota\ \sin 2\alpha\cos 2\beta\right]h_1(\gamma) \\ &+ \sin^2\iota\ \sin 2\beta\ h_2(\gamma)\,. \end{aligned} \tag{1}$$

GW amplitudes for an astrophysical MCE

Specific case: z-axis of MCE points along the direction of the wave (Z-axis) and choose (x, y) to coincide with (X, Y). Then:

$$h_{xy} \equiv h_+ = \frac{\mathcal{A}}{R}\, \tau^{-2/3} \eta_{xy}\,,$$
$$\eta_{xy}(\gamma) = \frac{1}{2}\left[\left(\frac{2}{9}\right)^{4/3\sin\gamma} - \left(\frac{2}{9}\right)^{4/3\sin(\gamma+\frac{2\pi}{3})}\right].$$

$\eta_{xy} \sim 1$ and for typical numbers:

$$h_{xy} \sim 4.57 \times 10^{-24} \left(\frac{M}{50\ M_\odot}\right)^{5/3} \left(\frac{R}{\mathrm{Gpc}}\right)^{-1} \eta_{xy}\ \tau^{-2/3}$$

GW strain averaged over uniform distribution of orientations of MCE:

$$h_{\mathrm{rms}}(\tau) \simeq 4.57 \times 10^{-24} \left(\frac{M}{50\ M_\odot}\right)^{5/3} \left(\frac{R}{\mathrm{Gpc}}\right)^{-1} \eta_{\mathrm{rms}}\ \tau^{-2/3}$$

where,

$$\eta_{\mathrm{rms}}(\gamma) = \frac{2}{\sqrt{15}}[\eta_{xy}^2 + \eta_{yz}^2 + \eta_{zx}^2]^{\frac{1}{2}}$$

Signal to Noise Ratio

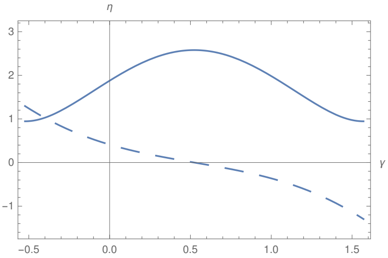

Figure: The dashed curve shows η_{xy} corresponding to the (x, y) axes and the continuous curve shows $\eta_{\rm rms}$ uniformly averaged over all orientations as a function of γ where $-\pi/6 \leq \gamma \leq \pi/2$. Since both η_{xy} and $\eta_{\rm rms}$ are shown in the same plot we have labelled the vertical axis by just η.

For zero detuned high power PSD of aLIGO:

$$\begin{aligned}
\rho(\gamma) &= 2\left[\int_{f_{\rm lower}}^{f_{\rm upper}} df \; \frac{|\tilde{h}_{\rm rms}(f;\gamma)|^2}{S_h(f)}\right]^{\frac{1}{2}} \\
&\sim 14.36 \times \left(\frac{M}{50\ M_\odot}\right)^{5/3} \left(\frac{R}{\rm Gpc}\right)^{-1} \eta_{\rm rms}(\gamma)\,.
\end{aligned}$$

Typically $\eta_{\rm rms} \sim 2$ with a maximum of $\sim 2.58 \longrightarrow \rho \sim 37$

Concluding Remarks

- Above SNRs computed for most favourable detector orientation but only for a single aLIGO at design sensitivity. More detectors will improve this number. For a uniform distrbution of sources and one aLIGO, $\rho_{\mathrm{average}} \sim 15 \implies$ **confident detection**.
- Superposition of unresolved MCEs could produce a stochastic GW background. For typical parameters $\Omega_{GW}(f) \sim 10^{-12}$ at $f \sim 10$ Hz for an MCE which lasts 1000 sec.
- Noise background will have to be estimated by time slides - Analysis simpler because only one template required for this simple model of MCE.
- Detecting another GW source would make the field of GW astronomy richer.
- MCEs provide an interesting test for cosmology. Their detection or otherwise will tell us if QSSC is viable.
- LIGO-India will provide an independent measurement - estimate orientation of MCE through polarisation, improve SNR etc.