2. Surprises in the Theory of Gravitation Waves

Goal: Illustrate through examples

(A) Some common misconceptions (notion of "Transverse Traceless modes" even in linearized gravity);
(B) Explain an unforeseen interplay between radiative and Coulombic aspects of electromagnetic and gravitational fields that complicates the expressions of angular momentum carried by these waves;
(C) Discuss the conceptual setting for a strong field test of GR that uses expressions of energy momentum and angular momentum carried by gravitational waves;
(D) Present some examples of how we can learn about quantum gravity from gravitational waves.

Parts (A) & (B) of this material appeared in a number of papers written jointly with Beatrice Bonga and Aruna Kesavan in 2016-17, where details can be found.

2.A. Two notions of Transverse Traceless Modes

Minkowski space-time (M, η_{ab}); spatial metric $q_{ab} = \eta_{ab} + t_a t_b$; $D_a q_{bc} = 0$.
Linear gravitational perturbation $\mathbf{h}_{ab}$; its spatial projection: h_{ab} (so $h_{ab}t^b = 0$).

- TT-Decomposition: $h_{ab} = \frac{1}{3} h q_{ab} + (D_a D_b s - \frac{1}{3} D^2 s) + D_{(a} V^T_{b)} + h^{\rm TT}_{ab}$

$h^{\rm TT}_{ab}$ fully gauge invariant but its extraction from h_{ab} is very non-local in physical space.
Local in the momentum space: In terms of the triad $\hat{k}^a, m^a(\vec{k}), \bar{m}^a(\vec{k})$ with
$\hat{k}^a \hat{k}_a = m^a(\vec{k}) \bar{m}_a(\vec{k}) = 1$; all other scalar products zero;

$$h^{\rm TT}_{ab}(\vec{x}, t) = \left[(2\pi)^{-\frac{3}{2}} \int d^3\vec{k}\; e^{i\vec{k}\cdot\vec{x}}\, \alpha(\vec{k}, t) m_a(\vec{k}) m_b(\vec{k})\right] + \text{CC}$$

This TT-notion is widely used in cosmology and perturbative QFT for describing gravitons.
(Section 35.4 of MTW; Box 5.7 in Poisson-Will; ...)

- Second TT notion: In a later section (36.10 of MTW; Chapter 11 of PW, ...) while considering gravitational waves from compact objects, one introduces another notion using a 'projector' $P_a^m := (q_a^m - \hat{r}_a \hat{r}^m)$ into a 2-sphere in physical space :

$$h^{TT}_{ab} = (P_a^m P_b^n - \tfrac{1}{2} P_{ab} P^{mn}) h_{mn} \quad \text{so that}$$

$$h^{TT}_{ab}(\vec{x}, t) = \left[\beta(\vec{x}, t) m_a(\vec{x}) m_b(\vec{x})\right]$$

where $m^a(\vec{x})$ is now the 'angular basis vector in the physical space. This $h^{\rm TT}_{ab}$ is extracted locally in physical space. Not gauge invariant!

$h_{ab}^{\rm TT}$ versus $h_{ab}^{\rm tt}$

• The Two notions are very different! Incorrect to use the same term and the same symbol $h_{ab}^{\rm TT}$. (Racz, AA & Bonga) So, let us use:

$$h_{ab}^{\rm TT}(\vec{x},t) = \left[(2\pi)^{-\frac{3}{2}} \int d^3\vec{k}\; e^{i\vec{k}\cdot\vec{x}}\, \alpha(\vec{k},t) m_a(\vec{k}) m_b(\vec{k})\right] \;+\; \rm CC$$

and,

$$h_{ab}^{\rm tt}(\vec{x},t) = \left[\beta(\vec{x},t) m_a(\vec{x}) m_b(\vec{x})\right] + \rm CC$$

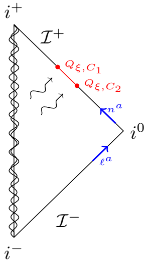
$h_{ab}^{\rm TT}(\vec{x},t)$ is fully gauge invariant. All 5 components non-zero even in the radiation zone; there are relations between them. By contrast: only the leading, $1/r$ part of $h_{ab}^{\rm tt}(\vec{x},t)$ is gauge invariant. It has only 2 components.

• Why did identifying these two entirely different notions not cause any problems? Can one use either of them to calculate energy, momentum, angular momentum carried by gravitational waves?

• To answer these questions, one has to consider the radiation zone, or, equivalently, null infinity $\mathcal{I}^+$. The analysis turns out to be much more subtle than one might have expected because fields can diverge at $\mathcal{I}^+$ unless gauge issues are carefully sorted out.

Relation between h_{ab}^{TT} and h_{ab}^{tt} at $\mathcal{I}^+$

- To analyze radiation, it is convenient to use the $u := t - r,\ r, \theta, \varphi$ coordinates and go to infinity ($r \to \infty$) along $u = u_0$. Limiting fields evaluated at the null boundary $\mathcal{I}^+$ coordinatized by $\bar{x} \equiv (u, \theta, \varphi)$ (and $r = \infty$, or, $\Omega = 1/r = 0$).



Set $\underline{\mathrm{h}}_{22}^{o}(\bar{x}) = h_{ab}^{\mathrm{TT}} m^a(\bar{x})\, m^b(\bar{x})|_{\mathcal{I}^+}$ & $\underset{\sim}{h}{}_{22}^{o}(\bar{x}) = h_{ab}^{\mathrm{tt}} m^a(\bar{x})\, m^b(\bar{x})|_{\mathcal{I}^+}$
Then, careful analysis shows: $\underline{\mathrm{h}}_{22}^{o} - \underset{\sim}{h}{}_{22}^{o} = f(\theta, \phi)$; The two strain tensors differ by a non-dynamical function at $\mathcal{I}^+$.

- Hence both serve as potentials for the radiation field captured in the component of Ψ_4^o of the Weyl tensor: $\Psi_4^o = \ddot{\underline{\mathrm{h}}}_{22} = \ddot{\underset{\sim}{h}}_{22}$. Flux of energy & momentum carried by gravitational waves across $\mathcal{I}^+$ can be calculated using $\dot{\underline{\mathrm{h}}}{}_{22}^{o}(\bar{x}) = \dot{\underset{\sim}{h}}{}_{22}^{o}(\bar{x})$. This is in large part why the erroneous identification did not cause any glaring problem!

- However, there is a difference: h_{ab}^{TT} has 5 components at $\mathcal{I}^+$ and they carry information also about the "Coulombic" part of the field that falls off faster in $1/r$ expansion than the radiative part. h_{ab}^{tt} has by construction only 2 components and therefore does not carry the "coulombic" information. This has a surprising consequence for angular momentum carried by gravitational waves!

2.B The Angular Momentum Surprise

• The main point can be readily seen using the familiar electromagnetic waves in Minkowski space! The analog of h_{ab} is the vector potential A_a. We again have $A_a^{\rm T}$ (which is non-local in space) and $A_a^{\rm t}$ (which is local in space). In gravity $h_{ab}^{\rm tt}$ much easier to compute than $h_{ab}^{\rm TT}$ and therefore widely employed in books and reviews. In the Maxwell theory, $A_a^{\rm t}$ is much easier to compute than $A_a^{\rm T}$.

• We are now led to set $\underline{\mathsf{A}}_2(\bar{x}) = A_a^{\rm T} m^a|_{\mathcal{I}^+}$ and $\underset{\sim}{A}_2(\bar{x}) = A_a^{\rm t} m^a|_{\mathcal{I}^+}$,
Again $\underline{\mathsf{A}}_2 - \underset{\sim}{A}_2 = f(\theta, \phi)$ is non-dynamical on $\mathcal{I}^+$, and the radiation Maxwell field $\Phi_2(\bar{x})$ can be expressed using either: $\Phi_2(\bar{x}) = \dot{\underline{\mathsf{A}}}_2(\bar{x}) = \dot{\underset{\sim}{A}}_2(\bar{x})$. Using the stress-energy tensor, one can calculate the energy flux $\mathcal{E}[C]$ across a cross-section of $\mathcal{I}^+$:

$$\mathcal{E}[C] = 2 \oint_C |\dot{\underline{\mathsf{A}}}_2|^2 \, \mathrm{d}^2 S = 2 \oint_C |\dot{\underset{\sim}{A}}_2|^2 \, \mathrm{d}^2 S \qquad .$$

Similarly for 3-momentum. To explore radiative aspects of the Maxwell field at $\mathcal{I}^+$ we can use either $A_a^{\rm T}$ or $A_a^{\rm t}$.

• While the radiation part of the Maxwell field Φ_2^o falls off as $1/r$ near $\mathcal{I}^+$, the Coulombic part Φ_1^o falls off as $1/r^2$. It is captured in $A_a^{\rm T}$, but not in $A_a^{\rm T}$. Complete parallel to the gravitational case.

Two notions of Transversality: Distinction Matters!

- Angular momentum: On $\mathcal{I}^+$, each rotational Killing field is of the type $R^a = R(\theta,\varphi)m^a + \mathrm{CC}$, (with $\bar{\eth}R(\theta,\varphi) = 0$). Using the stress-energy tensor, one can Angular momentum carried by electromagnetic waves across a cross-section of $\mathcal{I}^+$ is given by:

$$\mathcal{J}[C] = \sqrt{2}\oint_C \mathrm{Re}(\bar{\Phi}_2^o \Phi_1^o\, R)\ \mathrm{d}^2S\,.$$

In presence of sources, $\mathcal{J}[C]$ can be expressed using A_a^{T} that knows about the Coulombic field Φ_1^o, but not from A_a^{t}! There is an interesting but subtle interplay between radiative and Coulombic aspects.

- Surprise: Once we are in the radiation zone (or near $\mathcal{I}^+$) fields satisfy source-free equations. One generally thinks that the physical properties would not care whether the waves are globally source-free or were produced by sources. This expectation turns out to be incorrect for angular momentum because of a subtle interplay between the radiative and Coulombic parts.

In the gravitational case, radiative part is captured both in h_{ab}^{TT} and in h_{ab}^{tt}. But the Coulombic part is registered only in h_{ab}^{TT}.

- Therefore, in presence of sources, one would likely not be able to express angular momentum carried by gravitational waves using the more widely used h_{ab}^{tt}!

The issue was recently analyzed in a recent paper by Bonga and Poisson in the PN framework. In the full non-linear theory, the issue is being investigated.

2.C Testing the non-linear regime of GR

- A natural avenue would be to use observations to compare and contrast predictions of GR with those of alternate theories. But with a few exceptions (like Brans-Dicke theory which is under stress from other tests) we do not yet have reliable/believable (consistent, admitting initial value formulation for numerical simulations, ...) candidates that are 'worthy' of such systematic and detailed investigations.

- A conceptual setting for testing GR w/o reference to specific alternate theories. We assume that the PN and quasi-normal ringing predicted by GR is correct but allow for the possibility that in the full non-linear, strong field, dynamical regime, GR may be incorrect. Test the expressions of fluxes of energy, momentum, angular momentum carried by gravitational waves in full, non-linear GR. (AA & Streubel)

(i) Use the observed inspiral phase and PN methods to calculate the total (ADM) energy, momentum and angular momentum of the system.
(ii) Use the observed quasi-normal ringing (together with sophisticated statistics methods) to determine the final energy momentum and angular momentum of the black hole.
(iii) Use the observed, full wave form to calculate energy, momentum and angular momentum carried by gravitational waves in the framework of *exact* general relativity.
Note: One does not use the full numerical simulation anywhere.

Question: Is the result of (iii), that uses full GR, in agreement with (i) and (ii)?

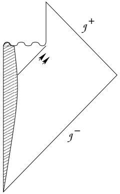
2.D Quantum Gravity and Gravitational waves

- Testing quantum gravity scenarios not a pie in the sky because some of the proposals proposals have been 'bold'. I will discuss a few examples.

(i) A serious proposal by Bekenstein and Mukhanov along the lines discussed e.g. by Foit & Keban arXiv: 1611.07009.
Motivation: BH entropy considerations suggest that the horizon area is quantized.
BM posit equal spacing: $A = \alpha N {\ell_{\rm pl}}^2$ with $\alpha \sim O(1)$. (Wheeler's It from Bit)

As energy is emitted, the horizon shrinks: Like atomic transitions. For a Spherical BH, if the transition occurs due to emission of n quanta we have $\Delta N = -n$. Therefore,
$\Delta A = \Delta\Big(4\pi(2GM)^2\Big) = -\alpha n G\hbar$ and $\Delta M = \hbar\omega_n$, whence $\omega_n = \frac{\alpha n}{32\pi GM}$.
For a Kerr black hole, we know A as a function $A(M, a)$ of mass and $a = J/M$. Using the BM proposal, we have: $\Delta A = -\alpha\hbar G n$; $\Delta M = -\hbar\omega_n$, $\Delta J = -\hbar m$ for some integer m. These equalities imply $\omega_n = \frac{1}{16\pi GM}\,\frac{\alpha n\sqrt{1-a^2}+8\pi a m}{1+\sqrt{1-a^2}}$
This is a different relation from quasi-normal modes.

A measurement of frequencies ω_n during the ringing (and of M, a) will either confirm this or (more likely) rule out the BM proposal. This would imply that the equal spacing of quantum horizon area eigenvalues is ruled out. In other approaches, the level spacing is not uniform; they will be favored. (In LQG, for example, they crowd exponentially for large A).



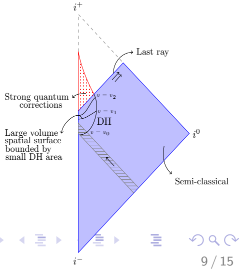
(ii) Significant deviations from GR outside horizons of large macroscopic BHs?

Motivation: The issue of unitarity and information recovery;
Giddings:1602.03622 Susskind; Heyden & Preskill; the Firewalls scenario (A. Almheiri, D. Marolf, J. Polchinski, D. Stanford and J. Sully); ...

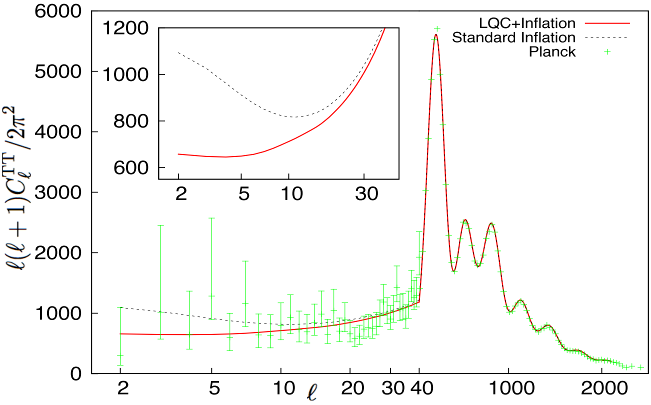
• Before GW observations, many researchers believed that the most plausible scenario is that quantum modifications of order $O(1)$ to GR occur outside the horizon over a distance scale of the horizon radius R. While there were no hard quantitative predictions, observations have ruled out such departures from GR. It may be worth looking at specific scenarios and make these observational constraints more precise.

In the more violent Firewall scenario, modifications only at a Planck scale near the horizon. This is *not* a prediction but 'a way out of contradictions' one arrives at from the assumptions one makes. As Param will discus, MM observations will severely constrain such violent effects.

• This does not imply that there is problem with unitarity!
Other proposals have been around which do not require such quantum modifications of GR near the horizon (e.g. AA & Bojowald). And in LQG, one now has detailed calculations showing that for macroscopic BHs, modifications outside the horizon would be negligibly small.
(. . . , AA, Olmedo, Singh)



(iii) Is there then any hope to see quantum gravity effects through gravitational waves? I think yes, but it will be through primordial gravitational waves. The early universe is an ideal place to look for quantum gravity in the sky!



Example: Power suppression at large angular scales in LQC . For CMB, the LQC power spectrum agrees with the standard BD power spectrum for $\ell \gtrsim 30$, but in LQC power is suppressed for $\ell \lesssim 30$. Thus, the LQC curve provides a better χ^2-fit to the full data. There are no new free parameters.

LQC predicts power suppression at large angular scales also for tensor modes (AA, Gupt) . But will probably take a while to verify/falsify this prediction.

Summary of Part 2

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2.E Gravitational waves in an accelerating universe

- There was considerable confusion on physical reality of gravitational waves in full general relativity because of diffeomorphism covariance of the theory.

- This confusion was finally dispelled by the Bondi-Sachs-Penrose et al framework that lies at the foundation of current understanding and is heavily used in numerical simulations. It was first presented by Bondi and Bergmann in GR3, Warsaw conference in 1962. But an interesting exchange followed:

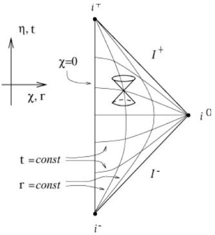
Weber: Why Asymptotically Minkowski (AM) boundary conditions rather than Friedmann type expanding universe boundary conditions?
Bergmann: Field is less than 2 years old . . . mathematically simplest case . . . purely psychological-historical . . .
Bondi: . . . I regret as much as you do that we haven't got to the point of doing the expanding universe.

- 56 years have passed since this exchange. The problem of describing the physics of gravitational waves in full non-linear GR in the accelerating universe we live in is still open!! Beyond asymptotic flatness, we still do not have the basic notions: Bondi news; Bondi energy-momentum and its local flux; . . . The radiation field Ψ_4^0 so heavily used in simulation used in LIGO templates ceases to be unambiguous!

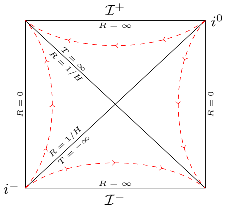
Accelerating universe & radiation

- Key Obstacle: To extract radiation, we have to go far away from sources. In the asymptotically flat case, curvature fades away and the task simplifies. But if the cosmological constant $\Lambda \neq 0$ then space-time curvature does not die even at infinity. Makes it difficult to tease out radiation in a coordinate invariant manner.

- And we do live in such a universe with $\Lambda > 0$! (Problems persist even if Λ were replaced by some form of dark energy causing accelerated expansion.) What do we even mean by gravitational waves in <u>*full*</u>, $\Lambda > 0$ GR? How do we extract physics of these waves? Today, the primary interest is conceptual since the numerical value of Λ is small. But in ~ 20 years these issues could well matter also in practice.



- With $\Lambda > 0$ the global structure of space-time changes drastically in the 'far zone' where radiation is extracted. Intuition from asymptotically flat case fails. Geometry $\to$ de Sitter rather than Minkowski. Already for Maxwell fields, while electromagnetic waves carry only positive energy in Minkowski space-time, they can have arbitrarily large (conserved) negative energy in deSitter.

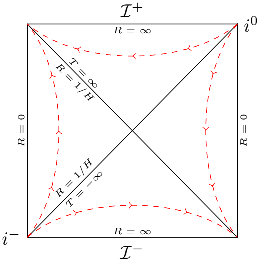


Current Status

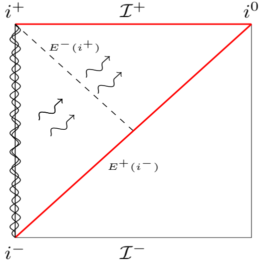
- Let us begin linearized gravitational waves. During 1916-18, Einstein used the first post-Minkowskian, first PN approximation, to obtain the celebrated quadrupole formula:

$$E_\xi = \frac{G}{8\pi} \int_{\mathcal{I}^+} (\dddot{Q}_{ab} \dddot{Q}^{ab}_{\mathrm{TT}})_{\mathrm{Ret}} \;\, \mathrm{d}\mathcal{I}^+$$

- The problem of extending it to the $\Lambda > 0$ case had remained open for a century because a host of unforeseen difficulties arise no matter how tiny Λ is! (Gravitational waves can carry arbitrarily large negative energy; $1/r$-expansions are not very useful; tail terms in retarded solutions; wave lengths increase as waves propagate making geometrical optics approximation invalid near $\mathcal{I}^+$; ...) Satisfactory generalization was obtained only in 1916!



Example: Positivity of energy. Energy across $\mathcal{I}^+$ can be negative only because ξ^a is past-directed along a portion of the cosmological horizon $E^+(i^-)$. But retarded solutions have no flux across $E^+(i^-)$. So the energy-flux through $\mathcal{I}^+$ is positive!



Thus, while the theory admits waves with negative de Sitter energy, such waves cannot be created by a time changing quadrupole.

- The final result provides very specific corrections to the quadrupole formula which bring out new aspects of relativistic gravity. Ex: We knew from the Raychaudhuri equation in cosmology that pressure contributes to gravitational attraction. We now learn that a time changing "pressure quadrupole" also sources gravity just like a time changing mass density quadrupole. We also have the detailed form of the corrections and know from first principles that these effects are negligible for current gravitational wave observatories. But recall: Effects of Einstein's quadrupole formula were also completely negligible at the time!

- Full general relativity: Only partial progress so far. It is embarrassing that we theorists do not have a gauge invariant characterization of gravitational waves in full GR with $\Lambda > 0$ yet! We have neither a good generalization of the radiation field Ψ_4^o nor of the Bondi news N_{ab} from $\mathcal{I}^+$ in AF space-times.

- Again, the current interest in this issue is mainly conceptual rather than observational. But it is unsettling that we know so little about full, general relativistic gravitational waves in the accelerating universe we live in.