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Quantum Field Theory

— Why, What and How

Springer

Preface

Paraphrasing the author of a quantum mechanics textbook, one might say that there is as much need for yet another book on QFT as there is for a New Revised Table of Integers! So let me explain why I am writing this book and how it will enrich the existing literature and fill a niche.

Standard textbooks in QFT go along a well-trodden path: Classical fields, their quantization, interactions and perturbation theory, tree-level computation of cross-sections using Feynman diagrams, divergences and renormalization as a way to handle them, etc. etc. Depending on the level of the textbook, the above topics will be discussed at different levels of sophistication and could be extended further towards e.g., gauge theories or the standard model. The calculational techniques you learn will be good enough to compute all the usual stuff in high energy physics, which is broadly the aim of most of these textbooks.

My interaction with bright graduate students, post-docs and theoretical physicists over the last three decades have made me realize some of the shortcomings of learning QFT exclusively along these lines. Let me list a few of them.

(1) Many smart students have asked me the following question: "If I know the classical action for a non-relativistic particle, I can construct the corresponding quantum theory using a path integral. I *do* know the classical action for a relativistic particle. Why can't I use it and construct the quantum theory of the relativistic particle and be done with it?"

This question goes to the core of QFT but — as far as I know — it is not answered directly in any of the existing textbooks. In fact, many high energy physicists (and at least one author of a QFT book) did not know that the exact, *non*quadratic, path integral for the relativistic particle *can* be calculated in closed form! Sure, every book pays lip service to why the single particle description is inadequate when you bring quantum theory and special relativity together, why the existence of antiparticles is a key new feature and all that. But this is done in a bit of hurry after which the author proceeds quickly to the classical field theory and its quantization! In other words, what these books do is to *start from fields and obtain particles* as their quanta rather than demonstrate that *if you start from a relativistic particle you will be led to the concept of fields.*

This is the first question I will address in Chapter 1. I will show, using the *exact evaluation of the relativistic path integral*, that if you start from the path integral quantization of the relativistic particle you will arrive at the notion of a field in a satisfactory manner. This route, from particles to fields (instead of from fields to particles) should be a welcome addition to the existing literature.

(2) The way renormalization is introduced in most of the textbooks is conceptually somewhat unsatisfactory. Many graduate students come away with the impression that renormalization is a trick to get meaningful answers out of divergent expressions, and do not understand clearly the distinction between regularization and renormalization. (The notable exceptions are students and textbooks with a condensed matter perspective, who do a better job in this regard.) Many authors feel that the Wilsonian perspective of quantum field theory is a bit too "advanced" for an introductory level course. In fact *it is not*, and students grasp it with a lot more ease (and with a lot less misunderstanding) than the more conventional approach. I will take the Wilsonian point of view as a backdrop right from the beginning, so that the student needs to re-learn very little as she progresses and will not have any fear for the so called "advanced" concepts.

(3) A closely related issue is the discussion of non-perturbative phenomena, which is *conspicuous by its absence* in almost all textbooks. This, in turn, makes students identify some concepts like renormalization too strongly with perturbation theory. Further, it creates difficulties for many students who are learning QFT as a prelude to specializing in areas where non-perturbative techniques are important. For example, students who want to work in gravitational physics, like QFT in external gravitational fields or some aspects of quantum gravity, find that the conventional perturbation theory approach — which is all that is emphasized in most QFT textbooks — leaves them rather inadequately prepared.

To remedy this, I will introduce some of the non-perturbative aspects of QFT (like, for example, particle production by external sources) *before* the students lose their innocence through learning perturbation theory and Feynman diagrams! Concepts like electromagnetic charge renormalization and running coupling constants, for example, will be introduced through the study of pair production in external electromagnetic fields, thereby divorcing them conceptually from the perturbation theory. Again, some of these topics (like e.g., the effective Lagrangian in QED) which are usually considered "advanced", are actually quite easy to grasp if introduced in an appropriate manner and early on. This will make the book useful for a wider class of readers whose interest may not be limited to just computing Feynman diagrams in the standard model.

(4) Most textbooks fail to do justice to several interesting and curious phenomena in field theory because of the rather rigid framework in which they operate. For example, the Davies-Unruh effect, which teaches us that concepts like "vacuum state" and "particle" can change in a non-inertial frame, is a beautiful result with far-reaching implications. Similarly, the Casimir effect is an excellent illustration of non-trivial consequences of free-field theory in appropriate circumstances. Most standard textbooks do not spend adequate amount of time discussing such fascinating topics which are yet to enter the mainstream of high energy physics. I will dip into such applications of QFT whenever possible, which should help students to develop a broader perspective of the subject.

Of course, a textbook like this is useless if, at the end of the day, the student cannot calculate anything! Rest assured that this book develops at adequate depth *all the standard techniques* of QFT as well. A student who reaches the last chapter would have computed the anomalous magnetic moment of the electron in QED, and would have worked out the two-loop renormalization of the $\lambda\phi^4$ theory. (You will find a description of the individual chapters of the book in the Chapter Highlights in pages v-vii, just after this Preface.)

In addition to nearly 80 Exercises sprinkled throughout the text in the marginal notes, I have also included 18 Problems (with solutions) at the end of the book. These vary significantly in their difficulty levels; some will require the student to fill in the details of the discussion in the text, some will illustrate additional concepts extending the ideas in the text, and some others will be applications of the results in the text in different contexts. This should make the book useful in self-study. So, by mastering the material in this book, the student will learn both the conceptual foundations as well as the computational techniques. The latter aspect is the theme of many of the excellent textbooks which are available and the student can supplement her education from any of them.

The readership for this book is very wide. Senior undergraduates, graduate students and researchers interested in high energy physics and quantum field theory will find this book very useful. (I expect the student to have some background in advanced quantum mechanics and special relativity in four-vector notation. A previous exposure to the nonrelativistic path integral will help, but is not essential.) The approach I have taken will also attract readership from people working in the interface of gravity and quantum theory, as well as in condensed matter theory. Further, the book can be easily adapted for courses in QFT of different durations.

The approach presented here has been tested out in the class room. I have been teaching selected topics in QFT for graduate students for about three decades in order to train them, by and large, to work in the area of interface between QFT and General Relativity. My lectures have covered many of the issues I have described above. The approach was well-appreciated and the students found it useful and enlightening. The feedback I got very often was that my course — taught at graduate student level — came a year too late!

Prompted by all these, in 2012, I gave a 50 hour course on QFT to *Masters-level* students in physics of the University of Pune. (In the Indian educational system, these are the students who will proceed to Ph.D graduate school in the next year.) The lectures were viedographed and made available from my institute website to a wider audience. (These are now available on YouTube; you can access a higher resolution video by sending a request to library@iucaa.ernet.in.) From the classroom feedback as well as from students who used the videos, I learnt that the course was a success. This book is an expanded version of the course.

Many people have contributed to this venture and I express my gratitude to them. To begin with, I thank the Physics Department of Pune University for giving me the opportunity to teach this course to their students in 2012, and many of these students for their valuable feedback. I thank Suprit Singh who did an excellent job of videographing the lectures. Sumanta Chakraborty and Hamsa Padmanabhan read through the entire first draft of the book and gave detailed comments. In addition, Swastik Bhattacharya, Sunu Engineer, Dawood Kothawala, Kinjalk Lochan, Aseem Paranjape, Sudipta Sarkar, Sandipan Sengupta, S. Shankaranarayanan and Tejinder Pal Singh read through several of the chapters and offered comments. I thank all of them. Angela Lahee of Springer initiated this project and helped me through its completion, displaying considerable initiative and accommodating my special formatting requirements involving marginal notes.

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T. Padmanabhan
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Chapter Highlights

1. **From Particles to Fields**

   The purpose of this chapter is to compute the path integral amplitude for the propagation of a free relativistic particle from the event x_1^i to the event x_2^i and demonstrate how the concept of a field emerges from this description. After introducing (i) the path integral amplitude and (ii) the standard Hamiltonian evolution in the case of a non-relativistic particle, we proceed to evaluate the propagator for a relativistic particle. An investigation of the structure of this propagator will *lead to* the concept of a field in a rather natural fashion. You will see how the standard unitary evolution, propagating forward in time, requires an infinite number of degrees of freedom for the proper description of (what you thought is) a *single* relativistic particle. In the process, you will also learn a host of useful techniques related to propagators, path integrals, analytic extension to imaginary time, etc. I will also clarify how the approach leads to the notion of the antiparticle, and why causality requires us to deal with the particle and antiparticle together.

2. **Disturbing the vacuum**

   The purpose of this — relatively short — chapter is to introduce you to the key aspect of QFT, viz., that particles can be created and destroyed. Using an external, classical scalar source $J(x)$, we obtain the propagator for a relativistic particle from general arguments related to the nature of creation and destruction events. The discussion then introduces functional techniques and shows how the notion of the field again arises, quite naturally, from the notion of particles which can be created or destroyed by external sources.

   By the end of the first two chapters, you would have firmly grasped how and why combining the principles of relativity and quantum theory *demands* a concept like the field (with an infinite number of degrees of freedom), and would have also mastered several mathematical techniques needed in QFT. These include path integrals, functional calculus, evaluation of operator determinants, analytic properties of propagators and the use of complex time methods.

3. **From fields to particles**

   Having shown in the first two chapters how the quantum theory of a relativistic particle naturally leads to the concept of fields, we next address the complementary issue of how fields lead to particles. After rapidly reviewing the action principle in classical mechanics, we make a seamless transition from mechanics to field theory. This is followed by a description of the (i) real and (ii) complex scalar fields and (iii) the electromagnetic field. Two key concepts in modern physics — spontaneous symmetry breaking and the notion of gauge fields — are introduced early on and in fact, the electromagnetic field will come in as a classical $U(1)$ gauge field.

   I then describe the quantization of real and complex scalar fields — which is fairly straightforward — and connect up with the ideas introduced in chapters 1 and 2. The discussion will compare the transition from particles to fields vis-a-vis from fields to particles, thereby strengthening conceptual understanding of both perspectives. The idea of particles arising as excitations of the fields naturally brings in the notion of Bogoliubov transformations. Using this, it is easy to understand the Unruh-Davies effect, viz., that the vacuum state in an inertial frame appears as a thermal state in a uniformly accelerated frame.

We next take up the detailed description of the quantization of the electromagnetic field. I do this first in the radiation gauge in order to get the physical results quickly and to explain the interaction of matter and radiation. This is followed by the covariant quantization of the gauge field which provides an opportunity to introduce the Fadeev-Popov technique in the simplest possible context, and to familiarize you with the issues that arise while quantizing a gauge field. Finally, I provide a detailed description of the Casimir effect which is used to introduce — among other things — the notion of dimensional regularization.

4. **Real life I: Interactions**

Having described the *free* quantum fields, we now turn to the description of interacting fields. The standard procedure in textbooks is to introduce perturbation theory, obtain the Feynman rules, calculate physical processes, and then introduce renormalization as a procedure to tackle the divergences in the perturbation theory, etc. For the reasons I described in the Preface, I think it is better to start from a non-perturbative approach, through the concept of effective action.

I will do this both for $\lambda\phi^4$ theory and for electromagnetic field coupled to a complex scalar field. In both the cases, one is led to the concept of renormalization group and that of running coupling constants. These, in turn, allow us to introduce the Wilsonian approach to QFT, which is probably the best language available to us today to understand QFT. The notion of effective action also leads to the Schwinger effect, viz., the production of charged particles by a strong electric field. This effect is non-analytic in the electromagnetic coupling constant, and hence cannot be obtained by perturbation theory.

After having discussed the non-perturbative effects, I turn to the standard perturbation theory for the $\lambda\phi^4$ case and obtain the usual Feynman diagrams (using functional integral techniques) and describe how various processes are calculated. This allows us to connect up themes like the effective Lagrangian and the running coupling constant from both perturbative and non-perturbative perspectives.

5. **Real life II: Fermions and QED**

Upto this point, I have avoided fermions in order to describe the issues of QFT in a simplified setting. This last chapter is devoted to the description of fermions and, in particular, QED. The Dirac equation is introduced in a slightly novel way through the relativistic square root $\sqrt{p^2} = \gamma^a p_a$, after discussing the corresponding nonrelativistic square root $\sqrt{\mathbf{p}^2} = \boldsymbol{\sigma} \cdot \mathbf{p}$ and the Pauli equation. Having obtained the Dirac equation, I describe the standard lore related to Dirac matrices and obtain the magnetic moment of the electron. I then proceed to discuss the quantization of the Dirac field, paying careful attention to the role of causality in fermionic field quantization. The path integral approach to fermionic fields is introduced through Grassmannians (which is developed to the extent required) and once again, we will begin with non-perturbative features like the Schwinger effect for electrons, before discussing perturbation theory and the Feynman rules in QED.

Finally, I provide a detailed discussion of the one loop QED and renormalization. This will allow, as an example, the computation of the anomalous magnetic moment of the electron, which many consider to be the greatest triumph of QED. The discussion of one loop diagrams in QED also allows the study of renormalization in the perturbative context and connect up the "running" of the electromagnetic coupling constant computed by the perturbative and non-perturbative techniques.

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