Nature of the singularities in higher dimensional Husain spacetime

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We generalize the previous studies of the spherically symmetric gravitational collapse in four dimensional spacetime to higher dimensions. It is found that the central singularities may be naked in higher dimensions but depend sensitively on the choices of the parameters. These naked singularities are found to be gravitationally strong that violate the cosmic censorship hypothesis.

Keywords: Cosmic censorship, naked singularity, higher dimensions, gravitational collapse.

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1. Introduction

It is widely believed that under physically reasonable conditions, a sufficiently massive collapsing body will undergo continual gravitational collapse, resulting in the formation of a gravitational singularity. This singularity may or may not be visible to a faraway observer. However, there is some debate as to whether such a singularity will be naked or covered by an event horizon. The cosmic censorship hypothesis (CCH) [1] states that the spacetime singularities arising in the gravitational collapse must be hidden behind the event horizons. In other words, the singularities forming in the gravitational collapse of a massive star are never naked. There are two versions of this hypothesis. The strong version suggests that no singularities are visible to any observers (i.e. local or faraway) while the weak CCH states that the singularities formed in the gravitational collapse are hidden inside the black holes and can not be seen by an observer at infinity.

Over the last 25 years or so, classical relativists are interested in formulating a proper provable version of CCH. Despite several attempts made by many researchers, this hypothesis remains unproven till date. Various models studied in the past years show that either naked singularities or black holes may form during the gravitational collapse. These models include radiation [2-6], dust [7-9], perfect fluid [10, 11], null strange quark fluid [12, 13] etc.

Over the last few years solutions of the Einstein equations in higher dimensions have come to play an important role in relativistic physics. The brane world scenario [14] suggests that our four dimensional world may be embedded in a higher dimensional spacetime. Although the extra dimension is not directly observable, but TeV-scale theory [15, 16] suggests that our universe may have large extra dimensions. It is believed that initially our universe may be having infinite dimensions, but then by dimensional reduction it got settled to four dimensional case to the lower energy level. To make such reduction possible, the dimensions should have much weight in any realistic model. In this sense, the higher dimensional case considered in this work has much importance.

From the view point of the CCH, one would like to know the effect of extra dimensions on the existence of a naked singularity. Would the examples of a naked singularity in four dimensions go over to HD or not? Does the CCH hold in higher dimensional spacetimes? Does the dimensionality play any role in the formation of a naked singularity? These are some important questions, which are to be studied in the higher dimensional gravitational physics. Many research papers on the higher dimensional collapse have appeared so far [17-22], which show that either naked singularities or black holes may form in the gravitational collapse depending upon the nature of the initial data. Hence, though the higher dimensional spacetimes are not so realistic, to study CCH, it now becomes essential to study the gravitational collapse of a matter in the higher dimensional spacetimes.

Husain solution of null fluid with $P = k\rho$ has been used to study the formation of a black hole with short hair [23] and can be considered as a generalization of Vaidya solution [24]. Recently, we have studied the gravitational collapse of the Husain solution in four dimensional spacetime [25] and found that this solution admits naked singularities under certain conditions on the mass function. Hence it would be interesting to see whether the higher dimensional collapse of Husain solution also leads to a naked singularity or not. In other words, in the present paper we generalize the earlier work in Ref.[25] to higher dimensions.

The paper is organized as follows: In section 2, we describe the five dimensional Husain spacetime and derive the radial null geodesics equations. In section 3, we investigate the formation and the nature of the singularities in higher dimensional asymptotically flat spacetimes. In section 4, we discuss the gravitational collapse of cosmological solution. Finally, the paper ends with concluding remarks.

2. Husain solution in five dimensional spacetime

General spherically symmetric line element in five dimensional (5D) spacetime is given by [26, 27]

$$ds^2 = -\left[1 - \frac{m(v,r)}{r^2}\right] dv^2 + 2dv\,dr$$
$$+ r^2\left(d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2\right), \quad (1)$$

where $m(v,r)$ is usually called mass function, and is related to gravitational energy within a given radius r. Null coordinate v represents the Eddington advanced time, in which r decreases towards the future along a ray v=constant, and ($d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1\, \sin^2\theta_2\, d\theta_3^2$) is a line element on unit 3-sphere.

Non-vanishing components of Einstein tensor are given by

$$G_0^0 = G_1^1 = -\frac{3m'}{2r^3}, \quad G_0^1 = \frac{3\dot{m}}{2r^3}, \quad G_2^2 = G_3^3 = G_4^4 = -\frac{m''}{2r^2}, \quad (2)$$

where

$$\{x^\mu\} = \{v, r, \theta_1, \theta_2, \theta_3\}, \ (\mu = 0, 1, 2, 3, 4),$$

and

$$\dot{m}(v,r) = \frac{\partial m}{\partial v}, \quad m' = \frac{\partial m}{\partial r}.$$

Combining Eq. (2) with the Einstein field equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad (3)$$

κ being gravitational constant, we find that the corresponding energy momentum tensor can be written in the form [24, 26, 28]

$$T_{\mu\nu} = T_{\mu\nu}^{(n)} + T_{\mu\nu}^{(m)}, \quad (4)$$

$$T_{\mu\nu}^{(n)} = \sigma\, l_\mu\, l_\nu, \quad (5)$$

$$T_{\mu\nu}^{(m)} = (\rho + P)(l_\mu \eta_\nu + l_\nu \eta_\mu) + P g_{\mu\nu}. \quad (6)$$

Combining Eqs. (2), (3) and (4) we can express the following quantities:

$$\sigma = \frac{3\dot{m}}{2\kappa r^3} \quad , \quad \rho = \frac{3m'}{2\kappa r^3} \quad , \quad P = -\frac{m''}{2\kappa r^2} \quad , \tag{7}$$

with l_μ and η_μ being two null vectors , and

$$l_\mu = \delta^0_\mu \quad , \quad \eta_\mu = \frac{1}{2}\left[1 - \frac{m(v,r)}{r^2}\right]\delta^0_\mu - \delta^1_\mu \quad ,$$

$$l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0 \quad , \quad l_\lambda \eta^\lambda = -1 \ . \tag{8}$$

Here ρ and P are energy density and pressure , while σ is the energy density of the Vaidya null radiation.

For these types of fluids, the energy conditions are given by [24, 28, 29]:

(a) The weak and strong energy conditions:

$$\sigma > 0 \quad , \quad \rho \geq 0 \quad , \quad P \geq 0 \ , \quad (\sigma \neq 0) \ . \tag{9}$$

(b) The dominant energy conditions:

$$\sigma > 0 \quad , \quad \rho \geq P \geq 0 \ , \quad (\sigma \neq 0) \ . \tag{10}$$

Following [26, 28] we define the mass function in five dimensional Husain solution:

$$m(v,r) = f(v) - \frac{g(v)}{(3k-1)r^{3k-1}} \quad , \quad k \neq \frac{1}{3} \ , \tag{11}$$

$$= f(v) + g(v)\ln r \quad , \qquad k = \frac{1}{3} \ ,$$

where $f(v)$ and $g(v)$ are arbitrary functions (which are restricted by the energy conditions).

For $k = 1/3$ weak and dominant energy conditions cannot be satisfied for all r because $\dot{m} = \dot{f}(v) + \dot{g}(v)\ln r$ can become negative for sufficiently small r [28]. Hence we'll not consider the case $k = 1/3$.

The physical situation is for $v < 0$, the spacetime is five dimensional Minkowskian, with $f(v) = 0$, $g(v) = 0$. The radiation is focused into a central singularity at $r = 0$, $v = 0$ of growing mass *f(v)* and *g(v).* At $v = T$, say, the radiation is turned off. For $v > T$, the exterior spacetime settles into a five dimensional Reissner Nordstrom solution. In case of four dimensional spacetime, Husain [28] defined the above solution by imposing the equation of state

$$P = k\rho \quad . \tag{12}$$

Substituting the mass function (11) into Eq. (7) we can find that

$$P = k \frac{3g(v)}{2\kappa\, r^{3(k+1)}} \quad = k\rho \ , \tag{13}$$

and

$$\sigma = \frac{3}{2\kappa r^3}\left[\dot{f}(v) - \frac{\dot{g}(v)}{(3k-1)\, r^{3k-1}} \right] . \tag{14}$$

Husain solution is characterized by the equation of state (12), $P = k\rho$, where $0 < k < 1$, due to which, we always have $\rho \geq P \geq 0$ and thus dominant energy conditions hold. It can be observed from Eqs. (11) , (13) and (14) that , to satisfy weak and strong energy conditions we must have $g(v) \geq 0$, and to ensure dominant energy conditions we would expect $\dot{m}(v,r) > 0$. (This has been discussed in details in Refs. [24, 28].

Inserting the expression for $m(v,r)$ from Eq. (11) into Eq. (1), we write the Husain metric in five dimensional spacetime:

$$ds^2 = -\left[1 - \frac{f(v)}{r^2} + \frac{g(v)}{(3k-1)\, r^{3k+1}}\right] dv^2 + 2dvdr$$
$$+ r^2\left(d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2\right). \tag{15}$$

To investigate the nature of the singularity which may form in the gravitational collapse, we need to consider the radial null geodesics defined by $ds^2 = 0$, taking $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$ into account.

Radial null geodesic equations (taking the null condition $K^a K_a = 0$ into account, where $K^a = dx^a / dk$ is the tangent vector to the geodesic) for the metric (1) are given by

$$\frac{dK^v}{dk} + \left(\frac{m}{r^3} - \frac{m'}{2r^2}\right)(K^v)^2 = 0 \ , \tag{16}$$

$$\frac{dK^r}{dk} + \left(\frac{\dot{m}}{2r^2} + \frac{m}{r^3} - \frac{m'}{2r^2} + \frac{mm'}{2r^4} - \frac{m^2}{r^5}\right)(K^v)^2 + \left(\frac{m'}{r^2} - \frac{2m}{r^3}\right)K^v K^r = 0. \tag{17}$$

Let

$$K^v = \frac{dv}{dk} = \frac{R(v,r)}{r} \ . \tag{18}$$

Using the null condition $K^a K_a = 0$, we get

$$K^r = \frac{R}{2r}\left(1 - \frac{m(v,r)}{r^2}\right), \tag{19}$$

where R satisfies the differential equation

$$\frac{dR}{dk} - \frac{R^2}{2r^2}\left(1 - \frac{3m}{r^2} + \frac{m'}{r}\right) = 0 \ . \tag{20}$$

The outermost boundary of the trapped region of the spacetime is known as an apparent horizon. For the five dimensional spacetime (15) the apparent horizon is given by

$$r^{3k+1} - f(v)\, r^{3k-1} + \frac{g(v)}{3k-1} = 0 \ . \tag{21}$$

The apparent horizon in the present case is defined as the outmost boundary of the trapped 3-sphere. The location of the trapped 3-sphere is the place where the outward normal of the surface r = const. is null. The apparent horizon is spacelike for $r > 0$ and $v > 0$.

If one considers the asymptotically flat solution (i.e. $k > 1/3$) and choose $k = 1/2$, $f(v) = \lambda v^2$, $g(v) = \mu v^{5/2}$ then the apparent horizon is given by the implicit relation

$$r^{5/2} - \lambda v^2 r^{1/2} + 2\mu v^{5/2} = 0 \ .$$

To find the relation between r and v, we use the "curve fitting technique" to get

$$r = \frac{v}{8} \ .$$

Thus the curve $r = v/8$ is an apparent horizon. Beyond the horizon, all solutions escape towards the future infinity and the singularity is naked, while inside the apparent horizon, all solutions reach the singularity $r = 0$ with finite v. The graph for this type of curve (i.e. $v = 8r$) is shown in Fig. 1. Similar type of graph can be found out for cosmological solution as well, (Fig. 2). (The graphs are given at the end of the paper).

Referring the definition of the asymptotically flat spacetime from [28], it can be observed that the spacetime (15) is asymptotically flat for $k > 1/3$ and is cosmological for $k < 1/3$. In the present work we analyze the gravitational collapse of the higher dimensional spacetimes in both the types of solutions (i.e. asymptotically flat as well as cosmological).

3. Gravitational collapse of the higher dimensional asymptotically flat spacetime.

For asymptotically flat spacetime we consider $1/3 < k < 1$.

Let us take $k = 1/2$. Then the mass function (11) becomes

$$m(v,r) = f(v) - \frac{2g(v)}{r^{1/2}}. \quad (22)$$

Substituting $k = 1/2$ into Eq. (15) we obtain the five dimensional asymptotically flat Husain solution

$$ds^2 = -\left[1 - \frac{f(v)}{r^2} + \frac{2g(v)}{r^{5/2}}\right] dv^2 + 2dvdr$$

$$+ r^2\left(d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2\right). \quad (23)$$

To investigate the nature of the singularity, we follow the method described in [6]. Roughly speaking, naked singularities are singularities that may be seen by the physically allowed observer, i.e. outgoing light rays starting from the singularity terminate at the singularity in the past. The central shell-focusing singularity (i.e. that occurring at $r = 0$) is naked, if the radial null geodesic equation admits one or more positive real root [30]. The outgoing radial null geodesic equation for the metric (23) is given by

$$\frac{dr}{dv} = \frac{1}{2}\left[1 - \frac{f(v)}{r^2} + \frac{2g(v)}{r^{5/2}}\right]. \quad (24)$$

The above equation does not yield analytic solution for general values of $f(v)$ and $g(v)$. However, if $f(v) \propto v^2$ and $g(v) \propto v^{5/2}$, the equation becomes homogeneous and can be solved in terms of the elementary functions [31].

Hence, let us choose $f(v) = \lambda v^2$ and $g(v) = \mu v^{5/2}$. Then the mass function (22) becomes

$$m(v,r) = \lambda v^2 - \frac{2\mu v^{5/2}}{r^{1/2}} \quad . \tag{25}$$

With these choices of the functions $f(v)$ and $g(v)$, the spacetime (23) becomes

$$ds^2 = -\left[1 - \frac{\lambda v^2}{r^2} + \frac{2\mu v^{5/2}}{r^{5/2}}\right] dv^2 + 2dvdr$$

$$+ r^2 (d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\; d\theta_3^2)\,. \tag{26}$$

It can be easily checked that the metric (26) is self-similar [32], admitting a homothetic killing vector ξ^a given by

$$\xi^a = v\frac{\partial}{\partial v} + r\frac{\partial}{\partial r} \;, \tag{27}$$

which satisfies

$$L_\xi\, g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab}\,, \tag{28}$$

where L denotes the Lie derivative.

It can be seen that $\xi^a K_a$ is constant along radial null geodesics, i.e.

$$\xi^a K_a = vK_v + rK_r = S\,, \tag{29}$$

where S is a constant.

Using the Eqs. (18), (19) and (25) into Eq. (29), we find the solution of the differential equation (20) as

$$R = \frac{2S}{2 - X + \lambda X^3 - 2\mu X^{7/2}} \;, \tag{30}$$

where we have defined $X = v/r$, and is known as a self-similarity variable.

To investigate the nature of the singularity, we need to consider the radial null geodesics defined by $ds^2 = 0$. Equation for the radial null geodesics for the metric (26) is given by

$$\frac{dv}{dr} = \frac{2}{1 - \frac{\lambda v^2}{r^2} + \frac{2\mu v^{5/2}}{r^{5/2}}} \quad . \tag{31}$$

It can be observed that the above differential equation has a singularity at $r = 0$, $v = 0$.

For the geodesic tangent to be uniquely defined and exist at this point we must have [30]

$$X_0 = \lim_{\substack{v \to 0 \\ r \to 0}} \frac{v}{r} = \lim_{\substack{v \to 0 \\ r \to 0}} \frac{dv}{dr} = \frac{2}{1 - \lambda X_0^2 + 2\mu X_0^{5/2}} \tag{32}$$

i.e.

$$2\mu X_0^{7/2} - \lambda X_0^3 + X_0 - 2 = 0 . \tag{33}$$

The above algebraic equation decides the nature of the singularity. If the above Equation has a real and positive root, then there exist future directed radial null geodesics originating from $r = 0$, $v = 0$. In this case the singularity will be naked. If the Eq. (33) has no real and positive root , then the singularity will be covered and the collapse proceeds to form a black hole.

Setting $X_0 = y^2$ in Eq. (33) we obtain

$$2\mu y^7 - \lambda y^6 + y^2 - 2 = 0 . \tag{34}$$

To analyze the nature of the root of the Eq. (34) the following rule in the *'theory of equations'* may be useful:

Every equation of odd degree has at least one real root whose sign is opposite to that of its last term, the coefficient of the first term being positive.

As in Eq. (34) the coefficient of the first term (i.e. 2μ) is positive and the last term is negative, the equation must have at least one positive root.

In particular, if we take $\lambda = 0.01$ and $\mu = 0.001$ then one of the roots of Eq. (34) is $y = 1.4362$,which on inserting in $X_0 = y^2$, gives $X_0 = 2.0625$. Thus we have obtained a real and positive root to the Eq. (33), which ensures that the singularity is naked. Further, if we set $\mu = 0$ [24] then the spacetime (26) reduces to five dimensional Vaidya spacetime and it has been shown in [33] that this solution admits naked singularity if $\lambda < 1/27$.

Strength of the naked singularity

The main importance of determining the strength of the singularity is due to the fact that the cosmic censorship hypothesis (CCH) does not need to rule out the possibility of the occurrence of the weak naked singularity [34].

A singularity is said to be strong if the collapsing objects do get crushed to a zero volume at the singularity, and a weak singularity if they do not. If the singularity is not strong, then it may not be considered as a physically realistic singularity.

Following Clarke and Krolak [35], a sufficient condition for a singularity to be strong in the sense of Tipler [36], is that, at least along one radial null geodesic (with affine parameter k) we must have

$$\lim_{k\to 0} k^2 \psi = \lim_{k\to 0} k^2 R_{ab} K^a K^b > 0 , \tag{35}$$

where K^a is the tangent to the null geodesics and R_{ab} is the Ricci tensor.
Using Eqs. (18) and (19) we obtain

$$k^2 R_{ab} K^a K^b = k^2 \left[\frac{3\dot{m}}{2r^3} \left(K^v \right)^2 \right] \tag{36}$$

$$= X \left[3\lambda - \frac{15}{2} \mu X^{1/2} \right] \left(\frac{kR}{r^2} \right)^2 . \tag{37}$$

Using the fact that as the singularity is approached, $k \to 0$, $r \to 0$, $X \to X_0$, and using L'Hospital's rule , we find that

$$\lim_{k\to 0} \frac{kR}{r^2} = \frac{1}{1 - \lambda X_0^2 + 2\mu X_0^{5/2}} . \tag{38}$$

Inserting the above equation into Eq. (37), we obtain

$$\lim_{k\to 0} k^2 R_{ab} K^a K^b = \frac{X_0\left(3\lambda - \frac{15}{2}\mu X_0^{1/2}\right)}{\left(1-\lambda X_0^2 + 2\mu X_0^{5/2}\right)^2} . \qquad (39)$$

Thus the singularity will be strong if

$$3\lambda - \frac{15}{2}\mu X_0^{1/2} > 0 .$$

For our particular case (i.e. $\lambda = 0.01$, $\mu = 0.001$, $X_0 = 2.0625$) we find that

$$3\lambda - \frac{15}{2}\mu X_0^{1/2} > 0 .$$

Thus the Clarke and Krolak condition for the strong curvature singularity is satisfied, hence the naked singularity arising in this spacetime is gravitationally strong.

4. Gravitational collapse of higher dimensional cosmological solution

The spacetime (15) will become cosmological for $k < 1/3$. Hence let us take $k = 1/5$. To get analytical solution chose $f(v) = \lambda v^2$ and $g(v) = \mu v^{8/5}$. With these choices, the mass function (11) becomes

$$m(v,r) = \lambda v^2 + \frac{5}{2}\mu v^{8/5} r^{2/5} . \qquad (40)$$

Using the above cosmological solution, the higher dimensional cosmological Husain solution can be written as

$$ds^2 = -\left[1 - \frac{\lambda v^2}{r^2} - \frac{5}{2}\frac{\mu v^{8/5}}{r^{8/5}}\right] dv^2 + 2dvdr$$

$$+ r^2(d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2) . \qquad (41)$$

It can be checked that the spacetime (41) is self-similar admitting a homothetic killing vector field ξ^a defined by Eq. (29). For the spacetime (41), the outgoing radial null geodesics must satisfy the null condition

$$\frac{dv}{dr} = \frac{2}{1 - \dfrac{\lambda v^2}{r^2} - \dfrac{5}{2}\dfrac{\mu v^{8/5}}{r^{8/5}}} \quad . \tag{42}$$

In order to determine the nature of the singularity at $r = 0$, $v = 0$, we let

$$X_0 = \lim_{\substack{v\to 0\\ r\to 0}} \frac{v}{r} = \lim_{\substack{v\to 0\\ r\to 0}} \frac{dv}{dr} = \frac{2}{1 - \lambda X_0^2 - \frac{5}{2}\mu X_0^{8/5}} \tag{43}$$

i.e.

$$\lambda X_0^3 + \frac{5}{2}\mu X_0^{13/5} - X_0 + 2 = 0. \tag{44}$$

Setting $X_0 = y^5$, we obtain

$$\lambda y^{15} + \frac{5}{2}\mu y^{13} - y^5 + 2 = 0 \ . \tag{45}$$

In particular for $\lambda = 0.01$ and $\mu = 0.001$, we find, using numerical methods that the Eq.(44) has a solution $X_0 = 2.1075$, which shows that the central singularity arising in the higher dimensional cosmological Husain spacetime is naked.

Further, following the method discussed in the previous section it can be checked that

$$\lim_{k\to 0} k^2 \psi = \lim_{k\to 0} k^2 R_{ab} K^a K^b$$

$$= \frac{X_0\left(3\lambda + 2\mu X_0^{-2/5}\right)}{\left(1 - \lambda X_0^2 - \frac{5}{2}\mu X_0^{8/5}\right)^2} > 0 \ , \tag{46}$$

which shows that the singularity arising in this case is also a gravitationally strong curvature singularity

5. Concluding remarks

We have generalized the earlier work given in Ref.[25] to higher dimensional Husain spacetime. We have considered both asymptotically flat as well as cosmological solutions for this study. It has been found that the results in 4D case can be extended to the five dimensions, essentially in the same manner, retaining the physical properties of the solution. We have examined the strength of the naked singularity using the criterion introduced by Tipler [36] and found that the naked singularity will be a strong curvature singularity depending on the appropriate choices of the values of the parameters λ and μ at $r = 0$. If we compare the apparent horizons between 4D and 5D cases, we find that the apparent horizon in 5D case gets somewhat reduced in comparison to that of 4D case. This might be the effect of the increase in the strength of gravity, as the gravitational force is directly proportional to the size of the extra dimensions [37].

The Kretschmann scalar for the five dimensional Husain spacetime is proportional to $r^{-6(k+1)}$ [26]. This shows that Kretschmann scalar diverges at $r = 0$, indicating that the singularity arising in this space time is a scalar polynomial curvature singularity. Along null geodesics, the Kretschmann scalar, energy density etc. diverge for $r \to 0$, so it is concluded that the singularity so formed at $v = r = 0$ is an ingoing-null naked singularity.

Formation of the naked singularities in the asymptotically flat as well as cosmological solutions indicate that, the condition of the asymptotically flatness of the spacetime does not play any fundamental role in the formation of the naked singularity in higher dimensional spacetimes as well.

In conclusion, we have shown that the extra dimensions cannot remove the formation of a naked singularity in the four dimensional collapse. Thus naked singularities do form in the higher dimensional spacetimes, that violates CCH.

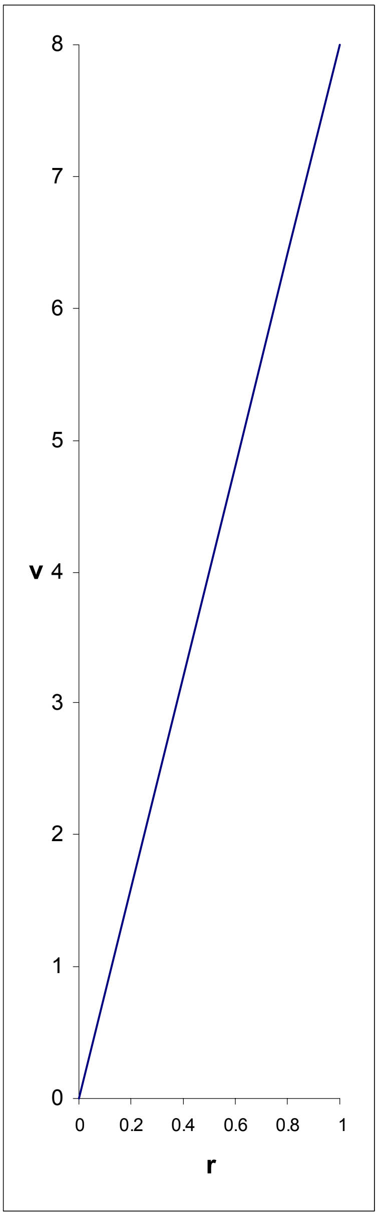

Fig. 1 : Apparent horizon in 5 – dimensional asymptotically flat Husain solution.
For $k = 1/2$,
$\boldsymbol{v = 8\,r}$.

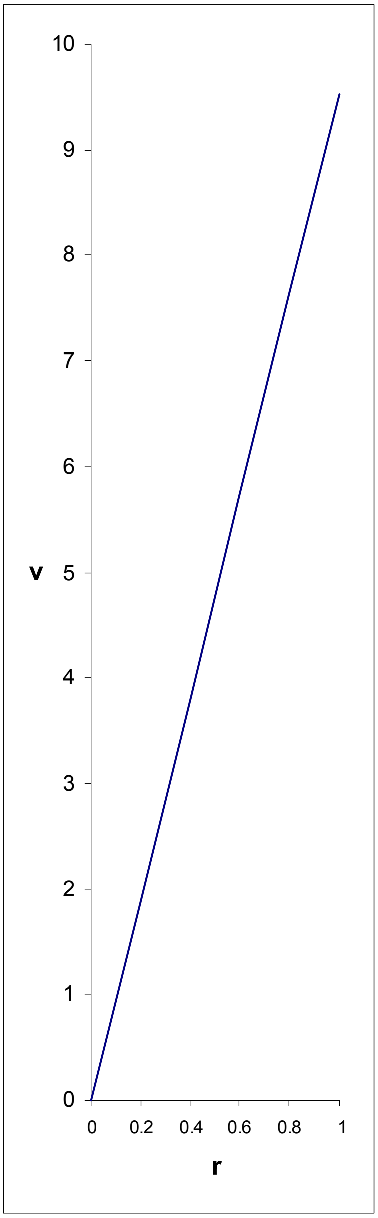

Fig. 2 : Apparent horizon in 5 – dimensional cosmological Husain solution
For $k = 1/5$,
$\boldsymbol{v = 9.5283\,r}$.

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