Five dimensional dust collapse with cosmological constant

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We study five dimensional spherical collapse of a inhomogeneous dust in presence of a positive cosmological constant. The general interior solutions, in the closed form, of the Einstein field equations, i.e., the 5D Tolman-Bondi-de Sitter, is obtained which in turn is matched to exterior 5D Scwarschild-de Sitter. It turns out that the collapse proceed in the same way as in the Minkowski background, i.e., the strong curvature naked singularities form and thus violate the cosmic censorship conjecture. A brief discussion on the causal structure singularities and horizons is also given.

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I. INTRODUCTION

The cosmological constant, conventionally denoted by the Greek letter Λ, is a parameter describing the energy density of the vacuum (empty space), and a potentially important contributor to the dynamical history of the universe. The original role of a non-zero Λ was to allow static homogeneous solutions to Einstein's equations in the presence of matter, turned out to be unnecessary when the expansion of the universe was discovered [1], and there have been a number of subsequent episodes in which a nonzero Λ was put forward as an explanation for a set of observations and later withdrawn when the observational case evaporated. But a non-zero Λ is still of interest, as observations made in the late 1990's of distance-redshift relations indicate that the universe is accelerating [2–4]. These observations can be explained very well by assuming a very small $\Lambda > 0$ in Einstein's equations. There are other possible causes of an accelerating universe, such as quintessence, but the cosmological constant is in most respects the most economical solution. If a Λ term must be restored to the Einstein equations, surprises may turn up in other physical applications of Einstein's equations as well. For example, Markovic and Shapiro [5] generalized the Oppenheimer-Snyder model (which describes the gravitational collapse of a spherical homogeneous dust ball initially at rest in exterior vacuum to a Schwarzschild black hole) taking into account the presence of a $\Lambda > 0$. They showed that Λ may affect the onset of collapse and decelerate the implosion initially. It was recently seen, by works of Susskind and others [6], a $\Lambda > 0$ has surprising consequences, such as a finite maximum entropy of the observable universe. The Markovic and Shapiro's [5] model was qualitatively generalized to the inhomogeneous and degenerate case by Cissoko *et al.* [7] and Lake [8, 9].

To understand the general collapse problem as well as nature of singularities, one would like to analyze exact solutions, preferably in close form, of Einstein equations. However the non-linearity of field equations makes them difficult, even in spherical symmetry. A model in which analytical treatment appears feasible is that of 5D Tolman-Model that describes gravitational collapse of spherically symmetric inhomogeneous dust in a 5D spacetime, since in this case the general exact solutions, for both marginally bound ($W(r) = 1$) and non-marginally bound ($W(r) \neq 1$), are possible. This is unlike the 4D case, where the corresponding solution is available in parametric form only. Hence, we shall restrict ourselves to the 5D spherically symmetric inhomogeneous dust collapse. Further, the 5D space-time is particularly more relevant because both 10D and 11D supergravity theories yield solutions where a 5D space-time results after dimensional reduction [10].

The aim of the paper is to extend the previous studies on the gravitational collapse of inhomogeneous dust in the presence of a $\Lambda > 0$ in the 5D space-time. First we derive general solutions, in closed form, for both marginally bound ($W(r) = 1$) and non-marginally bound ($W(r) \neq 1$) with $\Lambda > 0$. This is 5D analogous of 4D Tolman-Bondi-de Sitter solutions and for definiteness we shall call it 5D Tolman-Bondi-de Sitter solutions. This would be discussed in section II. We find that gravitational collapse of a 5D Tolman-Bondi-de Sitter spacetime gives rise to a naked shell-focusing singularity, providing an explicit counter-example to the cosmic censorship conjecture (CCC) [11]. This is is the subject of the section V which will be followed by a discussion. We discuss Darmois junction conditions between static and non-static space-times in section III. A brief discussion on apparent horizons formation 5D Tolman-Bondi-de Sitter solutions is presented in section IV.

We have used units which fix the speed of light and the gravitational constant via $8\pi G = c^4 = 1$.

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II. 5D TOLMAN-BONDI-DE SITTER SOLUTIONS

The standard 4D Tolman-Bondi solution [12] represents an interior of a collapsing inhomogeneous dust sphere. The solution we seek is - collapse of a spherical dust with positive Λ. The metric for the $5D$ case, in comoving coordinates coordinates, assumes the form [13–15]:

$$ds^2 = -dt^2 + X(t,r)^2 dr^2 + R(t,r)^2 d\Omega^2, \tag{1}$$

where

$$d\Omega^2 = d\theta^2 + \sin(\theta)^2 d\phi^2 + \sin(\theta)^2 \sin(\phi)^2 d\psi^2, \tag{2}$$

together with the stress-energy tensor for dust:

$$T_{ab} = \zeta(t,r)\delta^t_a \delta^t_b, \tag{3}$$

where $u_a = \delta^a_t$ is the 5-dimensional velocity. The coordinate r is the co-moving radial coordinate, t is the proper time of freely falling shells, R is a function of t and r with $R > 0$ and X is also a function of t and r. With the metric (1), the Einstein equations are

$$\begin{aligned} G^0_0 &= \frac{3}{X^2}\left(\frac{R''}{R} - \frac{X'R'}{XR} + \frac{R'^2}{R^2}\right) \\ &\quad -3\left(\frac{\dot{R}^2}{R^2} + \frac{\dot{X}\dot{R}}{XR}\right) - \frac{3}{R^2} = \zeta - \Lambda, \end{aligned} \tag{4}$$

$$G^1_1 = \frac{3R'^2}{X^2R^2} - 3\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right) - \frac{3}{R^2} = -\Lambda, \tag{5}$$

$$\begin{aligned} G^2_2 &= G^3_3 = G^4_4 = \frac{2}{X^2}\left(\frac{R''}{R} - \frac{X'R'}{XR} - \frac{R'^2}{2R^2}\right) \\ &\quad -2\left(\frac{\ddot{R}}{R} + \frac{\dot{X}\dot{R}}{XR} + \frac{\dot{R}^2}{2R^2} + \frac{\ddot{X}}{2X}\right) - \frac{1}{R^2} = -\Lambda, \end{aligned} \tag{6}$$

$$G^0_1 = 3\frac{\dot{R}'}{R} - 3\frac{\dot{X}R'}{XR} = 0, \tag{7}$$

where an over-dot and prime denote the partial derivative with respect to t and r, respectively. Integration of Eq. (7) gives

$$X = \frac{Y'}{W}, \tag{8}$$

where $W = W(r)$ is an arbitrary function of r. From Eq. (5) and (8), we obtain

$$\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1 - W^2}{R^2} - \frac{\Lambda}{3} = 0, \tag{9}$$

which can be easily integrated to yield

$$\dot{R}^2 = W^2 - 1 + \frac{\mathcal{M}}{R^2} + \Lambda\frac{R^2}{6}. \tag{10}$$

Where $\mathcal{M} = \mathcal{M}(r)$ is an arbitrary function of r and referred to as mass function. Since in the present discussion we are concerned with gravitational collapse, we require that $\dot{R}(t,r) < 0$. Substituting Eqs. (8) and (10) into Eq. (5) we obtain

$$\mathcal{M}' = \frac{2}{3}\kappa\zeta R^3 R'. \tag{11}$$

Integrating Eq. (11) leads to

$$\mathcal{M}(r) = \frac{2}{3}\kappa \int \zeta R^3 dR, \tag{12}$$

where constant of integration is taken as zero since we want a finite distribution of matter at the origin $r = 0$. The function $\mathcal{M}$ must be positive, because $\mathcal{M} < 0$ implies the existence of negative mass. This can be seen from the mass function $m(t,r)$, which is given by

$$\begin{aligned} m(t,r) &= R^2\left(1 - g^{ab}R_{,a}R_{,b}\right) \\ &= R^2\left(1 - \frac{R'^2}{X^2} + \dot{R}^2\right). \end{aligned} \tag{13}$$

Using Eqs. (8) and (10) into Eq. (13) we get

$$m(t,r) = \mathcal{M}(r) + \frac{\Lambda}{6}R^4(r,t). \tag{14}$$

The quantity $\mathcal{M}(r)$ can be interpreted as energy due to the energy density $\zeta(t,r)$ given by Eq. (12), and since it is measured in a comoving frame, $\mathcal{M}$ is only r dependent.

a. Marginally bound case ($W(r) = 1$) Equation (10) has three types of solutions, namely, hyperbolic, parabolic and elliptic solutions depending on whether $W(r) > 0$, $W(r) = 0$ or $W(r) < 0$ respectively. We consider the case $\Lambda > 0$ and without loss of generality we consider here $W(r) = 1$ case. The condition $W(r) = 1$ and $\Lambda = 0$ is the marginally bound condition, limiting the situations where the shell is bounded from those it is unbounded. In the presence of a cosmological constant, the situation is more complex, and $W(r) = 1$ leads to an unbounded shell. Then with the condition $W(r) = 1$ we obtain from Eqs. (8) and (10),

$$\begin{aligned} R(t,r) &= \left(\frac{6\mathcal{M}}{\Lambda}\right)^{1/4} \sinh^{1/2}\alpha, \\ R'(t,r) &= \left(\frac{6\mathcal{M}}{\Lambda}\right)^{1/4} \Big[\frac{\mathcal{M}'}{4\mathcal{M}} \sinh\alpha \\ &\quad + \sqrt{\frac{\Lambda}{6}}t_0' \cosh\alpha\Big] \sinh^{-1/2}\alpha, \end{aligned} \tag{15}$$

where $\alpha = \alpha(t,r)$ is given by

$$\alpha(t,r) = \sqrt{\frac{2\Lambda}{3}}[t_0(r) - t] \tag{16}$$

and $t_0(r)$ is an arbitrary function of r. For $t = t_0(r)$ we have $R(t,r) = 0$ which is the time when the matter shell r = constant hits the physical singularity. The three arbitrary functions $\mathcal{M}(r)$, $W(r)$ and $t_0(r)$ completely specify the behavior of shell of radius r. It is possible to make an arbitrary relabelling of spherical dust shells by $r \to g(r)$, without loss of generality, we fix the labelling by requiring that, on the hypersurface $t = 0$, r coincides with the radius $R(0,r) = r$. This corresponds to the following choice of $t_0(r)$:

$$t_0(r) = \sqrt{\frac{3}{2\Lambda}}\ \sinh^{-1}\left[\sqrt{\frac{\Lambda}{6\mathcal{M}}}\ r^2\right]. \tag{17}$$

The central singularity occurs at $r = 0$, the corresponding time being $t = t_0(0) = 0$. We denote by $\rho(r)$ the initial density

$$\rho(r) = \zeta(0,r) = \frac{\mathcal{M}'}{r^2} \to \mathcal{M}(r) = \int \rho(r) r^2 dr. \tag{18}$$

It is easy to see that as $\Lambda \to 0$ the above solution reduces to the 5D Tolman-Bondi solutions.

$$\lim_{\Lambda\to 0} R(t,r) = \left[\sqrt{4\mathcal{M}}(t_0 - t)\right]^{1/2},$$
$$\lim_{\Lambda\to 0} R'(t,r) = \frac{\mathcal{M}'(t_0 - t) + 4\mathcal{M}t_0'}{\left[4\mathcal{M}^3(t_0 - t)^2\right]^{1/4}}. \tag{19}$$

b. Non-marginally bound case ($W(r) \neq 1$) The case $W(r) \neq 1$ is interesting in the sense that in the analogous 4D case the solutions can't be obtain in closed form. Integration Eq. (10) shows that the evolution of the dust shell in the case $W(r) \neq 1$ is given by

$$R(t,r) = \left[\left(\frac{6\mathcal{M}}{\Lambda} - \frac{9(W^2-1)^2}{\Lambda^2}\right)^{1/2} \sinh\alpha - \frac{3(W^2-1)^2}{\Lambda}\right]^{1/2} \tag{20}$$

where again the $\alpha = \alpha(t,r)$ is given by Eq. (16). For $W(r) = 1$, the marginally bound solutions Eq. (15)are recovered.

III. JUNCTION CONDITIONS

In order to study the gravitational collapse of a finite spherical body we have to match the solution along the time like surface at some $R = R_\Sigma$ to a suitable $5D$ exterior. We consider a spherical surface with its motion described by a time-like 4-surface Σ, which divides space-times into interior and exterior manifolds $\mathcal{V}_I$ and $\mathcal{V}_E$ Since the fluid is not radiating the exterior space-time to the Σ can be taken as $5D$ Schwarzschild-de Sitter space-time:

$$ds^2 = -\chi(\mathbf{r})dT^2 + \frac{1}{\chi(\mathbf{r})}d\mathbf{r}^2 + \mathbf{r}^2 d\Omega^2, \tag{21}$$

where

$$d\Omega^2 = d\theta^2 + \sin(\theta)^2 d\phi^2 + \sin(\theta)^2 \sin(\phi)^2 d\psi^2,$$

and

$$\chi(\mathbf{r}) = 1 - \frac{2M}{\mathbf{r}} - \frac{\Lambda}{3}\mathbf{r}^2.$$

In accordance with Darmois junction condition [7], we have to demand when approaching Σ in $\mathcal{V}_I$ and $\mathcal{V}_E$

$$(ds_-^2)_\Sigma = (ds_+^2)_\Sigma = (ds^2)_\Sigma, \tag{22}$$

where the subscript Σ means that the quantities are to be evaluated on Σ and let $K_{ij}^{\pm}$ is extrinsic curvature to Σ, defined by

$$K_{ij}^{\pm} = -n_\alpha^{\pm}\frac{\partial^2\chi_\pm^\alpha}{\partial\xi^i\partial\xi^j} - n_\alpha^{\pm}\Gamma_{\beta\gamma}^\alpha\frac{\partial\chi_\pm^\beta}{\partial\xi^i}\frac{\partial\chi_\pm^\gamma}{\partial\xi^j}, \tag{23}$$

and where $\Gamma_{\beta\gamma}^\alpha$ are Christoffel symbols, $n_\alpha^{\pm}$ the unit normal vectors to Σ, χ^α are the coordinates of the interior and exterior space-time and ξ^i are the coordinates that defines Σ. The intrinsic metric on the hypersurface $r = r_\Sigma$ is given by

$$ds^2 = -dt^2 + R^2(r_\Sigma, t)d\Omega^2, \tag{24}$$

with coordinates $\xi^a = (t,\ \theta,\ \phi,\ \psi)$. In this coordinate the surface Σ, being the boundary of the matter distribution, will have the equation

$$r - r_\Sigma = 0, \tag{25}$$

where r_Σ is a constant. The first fundamental form of Σ can be written as $g_{ij}d\xi^i d\xi^j$. Then the exterior metric, on Σ, becomes:

$$ds_\Sigma^2 = -\left[\chi(\mathbf{r}_\Sigma) - \frac{1}{\chi(\mathbf{r}_\Sigma)}\left(\frac{d\mathbf{r}_\Sigma}{dT}\right)^2\right] dT^2 + \mathbf{r}_\Sigma^2 d\Omega^2, \tag{26}$$

where we assume that the coefficient of $dT^2 > 0$ so that T is time like coordinate. From the first junction condition we obtain

$$\mathbf{r}_\Sigma = R(\mathbf{r}_\Sigma, t), \tag{27}$$

$$\left[\chi(\mathbf{r}_\Sigma) - \frac{1}{\chi(\mathbf{r}_\Sigma)}\left(\frac{d\mathbf{r}_\Sigma}{dT}\right)^2\right]^{1/2} dT = dt. \tag{28}$$

The non-vanishing components of extrinsic curvature $K_{ij}^{\pm}$ of Σ can be calculated and the result is

$$K_{T\,T}^{+} = \left[\dot{\mathbf{r}}\ddot{T} - \dot{T}\ddot{\mathbf{r}} - \frac{\chi}{2}\frac{d\chi}{d\mathbf{r}}\dot{T}^3 + \frac{3}{2\chi}\frac{d\chi}{d\mathbf{r}}\dot{T}\dot{\mathbf{r}}^2\right]_\Sigma, \tag{29}$$
$$K_{\theta\,\theta}^{+} = \left[\chi\mathbf{r}\dot{T}\right]_\Sigma, \tag{30}$$
$$K_{t\,t}^{-} = 0, \tag{31}$$
$$K_{\theta\,\theta}^{-} = \left[\frac{RR'}{X}\right]_\Sigma. \tag{32}$$

With the help of Eqs. (22) - (32) and (10)

$$M = \mathcal{M}(r). \tag{33}$$

which can be interpreted as the total energy entrapped within the surface Σ [16]. Thus the junction conditions demand that the 5D Schwarschild mass M is given by Eq. (33).

IV. APPARENT HORIZON

The apparent horizon is formed when the boundary of trapped three spheres are formed. The apparent horizon is the solution of

$$g^{ab}R_{,a}R_{,b} = -\dot{R}^2 + \frac{R'^2}{X^2} = 0. \tag{34}$$

Upon using Eqs. (8) and (10), we have

$$\Lambda R^4 - 6R^2 + 6\mathcal{M} = 0. \tag{35}$$

For $\Lambda = 0$ we have the Schwarzschild horizon $R = \pm\sqrt{\mathcal{M}}$, and for $\mathcal{M} = 0$ we have the de Sitter horizon $R = \pm\sqrt{6/\Lambda}$. For $2\mathcal{M} < 3/\Lambda$ there are two horizons:

$$R_1^2 = \pm\sqrt{\frac{3}{\Lambda} + \frac{\sqrt{36 - 24\Lambda\mathcal{M}}}{2\Lambda}},$$
$$R_2^2 = \pm\sqrt{\frac{3}{\Lambda} - \frac{\sqrt{36 - 24\Lambda\mathcal{M}}}{2\Lambda}}. \tag{36}$$

For $\mathcal{M} = 0$ we have generalized cosmological horizon $R_1 = \pm\sqrt{6/\Lambda}$ otherwise generalized black hole horizon for $\Lambda \neq 0$. For $2\mathcal{M} > 3/\Lambda$ there are no horizons. The time at which apparent horizons are formed, from is

$$t_{AH} = t_0(r) - \sqrt{\frac{3}{2\Lambda}}\ \sinh^{-1}\left[\sqrt{\frac{\Lambda\,\mathcal{M}}{6}}\right], \tag{37}$$

In the limit $\Lambda = 0$ above equation reduces to,

$$t_{AH} = t_0 - \frac{\sqrt{\mathcal{M}}}{2}. \tag{38}$$

From Eq. (37), we have

$$\frac{R_n}{\mathcal{M}} = \cosh^2\alpha_n. \tag{39}$$

It is evident from Eq. (36) that $R_1 \geq R_2$, also from Eq. (39) $\alpha_1 \geq \alpha_2$ or $t_1 \geq t_2$, which means that the cosmological horizon always precedes the black hole horizon.

V. NATURE OF SINGULARITIES

The easiest way to detect a singularity in a space-time is to observe the divergence of some invariants of the Riemann tensor. The Kretschmann scalar ($\mathrm{K} = R_{abcd}R^{abcd}$, R_{abcd} the Riemann tensor) for the metric (1) reduces to

$$\mathrm{K} = 7\frac{F'^2}{R^6R'^2} - 36\frac{FF'}{R^7R'} + 72\frac{F^2}{R^8} + \frac{2}{3}\frac{\Lambda F'}{R'R^3} + \frac{10}{9}\Lambda^2 \tag{40}$$

and the Weyl scalar ($\mathrm{C} = C_{abcd}C^{abcd}$, C_{abcd} the Weyl tensor) takes the form:

$$\mathrm{C} = \frac{9}{2}\frac{F'^2}{R^6R'^2} - 36\frac{FF'}{R^7R'} + 72\frac{F^2}{R^8} \tag{41}$$

The Kretschmann scalar, Weyl scalar and energy density diverge at $t = t_0(r)$ indicating the presence of a scalar polynomial curvature singularity [17]. It has been shown [18] that Shell-crossing singularities are characterized by $R' = 0$ and $R > 0$. On the other hand the singularity at $R = 0$ is where all matter shells collapses to a zero physical radius and hence known as shell focussing singularity. We shall consider the case $t \geq t_0$. In the context of the Tolman-Bondi models the shell crossings are defined to be surfaces on which $R' = 0$ ($R > 0$) and where the density ζ diverges. A regular extremum in R along constant time slices may occur without causing a shell crossing, provided $\zeta(t, r)$ does not diverge. Where

$$\zeta(t, r) = \frac{3\mathcal{M}'}{2R^3R'}. \tag{42}$$

By Eq. (42), this implies $\mathcal{M}' = 0$ where ever $R' = 0$ and also that the surface $R' = 0$ remain at fixed R. Now Eq. (15) implies $t_0' = 0$. Thus the condition for a regular maximum in $R(t, r)$ is that $\mathcal{M}' = 0$, $t_0' = 0$ hold at the same R.

Christodoulou [19] pointed out in the $4D$ case that the non-central singularities are not naked. Hence, we shall confine our discussion to the central shell focusing singularity. It is known that, depending upon the inhomogeneity factor, the $4D$ Tolman-Bondi solutions admits a central shell focusing naked singularity in the sense that outgoing geodesics emanate from the singularity. Here we wish to investigate the similar situation in our 5D space-time. In what follows, we shall confine ourselves to the marginally bound case ($W(r) = 1$). We consider a class of models such that

$$\mathcal{M}(r) = \gamma r^2, \tag{43a}$$
$$t_0(r) = Br. \tag{43b}$$

This class of models for 4D space-time is discussed in [8, 20, 21]. The parameter B gives the inhomogeneity of the collapse. For $B = 0$ all shells collapse at the same time. For higher B the outer shells collapse much later than the central shell. We are interested in the causal structure of the space-time when the central shell collapses to the center ($R = 0$). The energy density at the singularity

$$\zeta = \frac{3\gamma}{r^2} \tag{44}$$

and the equation of general density becomes

$$\zeta = \frac{\Lambda}{\sinh^2\left[\sqrt{\frac{2\Lambda}{3}}(B-y)\,r\right]\left[1+\sqrt{\frac{2\Lambda}{3}}\,rB\,\coth\left[\sqrt{\frac{2\Lambda}{3}}(B-y)\,r\right]b\right]}. \tag{45}$$

As $t \to t_0(r)$ i.e. in approach to singularity, we have $\sinh\alpha \approx \alpha$ and $\coth\alpha \approx 1/\alpha$ then Eq. (45) reduces to

$$\zeta = \frac{3y^2}{2(B-y)(2B-y)t^2} = \frac{c(y)}{t^2}. \tag{46}$$

The nature (a naked singularity or a black hole) of the singularity can be characterized by the existence of radial null geodesics emerging from the singularity. The singularity is at least locally naked if there exist such geodesics, and if no such geodesics exist, it is a black hole. The critical direction is the Cauchy horizon. This is the first outgoing null geodesic emanating from $r = t = 0$. The Cauchy horizon of the space-time has $y = t/r = \text{const}$ [20–22]. The equation for outgoing null geodesics is

$$\frac{dt}{dr} = R' \tag{47}$$

Hence along the Cauchy horizon, we have

$$R' = y \tag{48}$$

and using Eqs. (48) and (15), with our choice of the scale, we obtain the following algebraic equation:

$$y^2\left(1-\frac{y}{B}\right) = 2B\sqrt{\gamma}\left[1-\frac{1}{2}\frac{y}{B}\right]^2. \tag{49}$$

To facilitate comparison with work in [13, 15], we introduce a relation between B and γ as,

$$B = \frac{1}{2\sqrt{\gamma}}, \tag{50}$$

then Eq. (49) can be written as

$$y^2\left(1-\frac{y}{B}\right) = \left[1-\frac{1}{2}\frac{y}{B}\right]^2. \tag{51}$$

This algebraic equation governs the behavior of the tangent vector near the singular point. The central shell focusing singularity is at least locally naked, if Eq. (51) admits one or more positive real roots. Hence in the absence of positive real roots, the collapse will always lead to a black hole. Thus, the occurrence of positive real roots implies that the strong CCC is violated, though not necessarily the weak CCC. If Eq. (49) has only one positive root, a single radial null geodesic would escape from the singularity, which amounts to a single wave front being emitted from the singularity and hence singularity would appear to be naked only, for an instant, to an asymptotic observer. A naked singularity forming in gravitational collapse could be physically significant if it is visible for a finite period of time, to an asymptotic observer, i.e., a family of geodesics must escape from the singularity. This happens only when Eq. (49) admits at least two positive real roots [23].

It can be shown that Eq. (51) has two positive roots if $B > B_c = 1.6651$. This is slightly higher than the analogous value, $B_c^4 = 1.56736$, in 5D. The corresponding Cauchy horizon evolves as $y = 0.78611$. Thus in 5D one needs higher inhomogeneity to produce naked singularity. For $B > B_c$, two solutions exist, the largest y gives the Cauchy horizon. Other solution is termed as self-similar horizon [20]. The results obtained here agree with the earlier work [22, 23]. Since these analysis were done in Minkowskian background or asymptotic flat setting ($\Lambda = 0$). As a result, one can claim that the Tolman-Bondi-de Sitter space-time has same singularity behavior as the Tolman-Bondi space-time.

The very fact the Eq. (51) is the same as that obtained in [13, 15] when the metric is asymptotically flat, i.e., when $\Lambda = 0$, *implies that the values of roots for the geodetic tangent and the condition for these values to be real and positive are the same as those obtained for the asymptotically flat situation in [13, 15]. As a result, the 5D Tolman-Bondi-de Sitter space-time has same singularity behavior as the 5D Tolman-Bondi space-time. Further, it is interesting to note that, as* $t \to t_0(r)$, *i.e., in approach singularity, we have* $\sinh\alpha \approx \alpha$ *and* $\coth \approx 1/\alpha$ *and, then Eq. (15) reduces to Eq. (19). Thus the 5D Tolman-Bondi-de Sitter solutions approach the Tolman-Bondi solutions as* $t \to t_0(r)$. *These results are consistent with in the analogous 4D dust collapse [24]* and radiation collapse [25].

As a result, the Tolman-Bondi-de Sitter space-time has same singularity behavior as the Tolman-Bondi space-time in both 4D and HD. Thus the final fate of collapsing inhomogeneous dust in de Sitter background is similar to that of collapsing inhomogeneous dust in Minkowskian background, as it should have been expected, since when $t \to t_0(r)$ ($R \to 0$) the cosmological term $\Lambda R^2/6$ is negligible. Thus we can assert that the asymptotic flatness is not a necessary ingredient for the formation of naked singularity for at least for a class of models considered.

It is known that the Tolman-Bondi metric ($\Lambda = 0$) in the 4D case is extensively used for studying the formation of naked singularities in spherical gravitational collapse. It has been found that Tolman-Bondi metric admit both naked singularities and black holes form depending upon the choice of initial data. Indeed, both analytical [18, 20, 22, 23] and numerical results [26] in dust indicate the critical behavior governing the formation of black holes or naked singularities. One can now safely assert that end state of 4D Tolman-Bondi collapse is now completely known in dependence of choice of initial data. A similar situation also occurs in 5D Tolman-Bondi collapse [13, 15]. Hence, we can conclude both naked singularities and black holes can form in the 5D spherical inhomogeneous dust collapse with a positive Λ.

VI. DISCUSSION

In the study of the Einstein equations in the 4D space-time several powerful mathematical tools were developed, based on the space-time symmetry, algebraical structure of space-time, internal symmetry and solution generation technique, global analysis, and so on. It would be interesting how to develop some of these methods to higher dimensional space-time. With this as motivation, plus the fact that exact solutions are always desirable and valuable, we have derived the exact spherically symmetric solution of inhomogeneous dust collapse in the presence of the cosmological constant in the five dimensional space-time. Although at first sight obtaining such a solution appears to be pure mathematical interest, further thought suggest opposite. We found general exact solutions, for both marginally bound ($W(r) = 1$) and non-marginally bound ($W \neq 1$), that describes gravitational collapse of spherically symmetric inhomogeneous dust in a 5D space-time. This is unlike the 4D case, where the corresponding solutions for non-marginally bound case, is available in parametric form only.

A gravitational collapse inevitably results in a space-time singularity, once the collapse has gone beyond a certain point, according to general relativity theory. The phrase *cosmic censorship* refers to two closely related conjectures about the nature of these space-time singularities, due to Roger Penrose [11]. The *weak* CCC states in essence says that gravitational collapse from regular initial conditions never creates a space-time singularity visible to distant observers. The idea here is that any singularity that forms must be hidden within a black hole. The *strong* CCC holds that any such singularity is never visible to any observer at all, even someone close to it. A *naked* space-time singularity would therefore be one that contradicts these conjectures.

The introduction of a cosmological constant changes this scenario in many ways. There are more apparent horizons instead of one. However, only two apparent horizon are physical, namely the black hole horizon and the cosmological horizon. Other results derived in [7] do carry over to HD space-time essentially with same physical behavior.

Our investigation establishes that the space-time is asymptotically flat or not does not make any difference to the occurrence of a naked singularity. This evident at least the in the class of models given by Eq. (43). Thus, the 5D Tolman-Bondi de Sitter metric admits both naked and covered singularities depending upon the choice of initial data and hence contradicts the weak cosmic censorship conjecture.

Finally, the result obtained would also be relevant in the context of superstring theory which is often said to be next "theory of everything", and for an interpretation of how critical behavior depends on the dimensionality of the space-time.

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[1] E. P. Hubble, Proc. Nat. Acad. Sci. U.S.A. **15**, 168, (1929).
[2] A. G. Reiss, *et al.* Astron. J. **116**, 1009 (1998).
[3] S. Perlmutter, *et al.* Nature **391**, 51 (1998); Ap. J. **517**, 565 (1999).
[4] V. Sahani and A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).
[5] D. Markovic and S. L. Shapiro, Phys. Rev. D **61**, 084029 (2000).
[6] Lisa Dyson, Matthew Kleban and Leonard Susskind, JHEP **0210**, 011 (2002).
[7] M. Cissoko, J. C. Fabris, J. Gariel, G. L. Denmat and N. O. Santos, Preprint gr-qc/9809057.
[8] K. Lake, Phys. Rev. D **62**, 027301 (2000).
[9] S. S. Deshingkar, S. Jhingan, A. Chamorro and P. S. Joshi, Phys. Rev. D **63**, 124005 (2001).
[10] J.J. Schwarz, Nucl. Phys. **B226**, 269 (1983).
[11] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969); in *General Relativity - an Einstein Centenary Volume*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979) pp 581-638.
[12] R. C. Tolman, Proc. Natl. Acad. Sci. USA. **20**, 169 (1934); H. Bondi, Mon. Not. R. Astron. Soc. **107**, 410 (1948).
[13] S. G. Ghosh and A. Banerjee, Int. J. Mod. Phys. D **12**, 639 (2003).
[14] A. Banerjee, A. Sil and S. Chatterjee, Astrophys. J. **422**, 681 (1994); A. Sil and S. Chatterjee, Gen. Relativ. Gravit. **26**, 999 (1994).
[15] S. G. Ghosh and A. Beesham, Phys. Rev. D **64**, 124005 (2001).
[16] M. E. Cahill and G. C. McVittie, J. Math. Phys. **11**, 1382 (1970).
[17] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time* Cambridge University Press, Cambridge,1973.
[18] R. P. A. C. Newman, Classical Quantum Gravity **3**, 527 (1986).
[19] D. Christodoulou, Commun. Math. Phys. **93**, 171 (1984).
[20] J. P. S. Lemos, Phys. Lett. A **158**, 271 (1991); *ibid.* Phys. Rev. Lett. **68**, 1447 (1992).
[21] A. Ori and T. Piran, Phys. Rev. D **42**,1068 (1990).

[22] B. Waugh and K. Lake, Phys. Rev. D **38**, 1315 (1988).
[23] I. H. Dwivedi and P. S. Joshi, Classical Quantum Gravity **9**, L69 (1992); P. S. Joshi and T. P. Singh, Gen. Relativ. Gravit. **27**, 921 (1995); *ibid.* Phys. Rev. D **51**, 6778 (1995).
[24] S. G. Ghosh Int. J. Mod. Phys. D **14**, 707 (2005).
[25] S. M. Wagh and S. D. Maharaj, Gen. Relativ. Gravit. **31**, 975 (1999).
[26] D. M. Eardley and L. Smarr, Phys. Rev. D **19**, 2239 (1979); D. M. Eardley, in *Gravitation in Astrophysics*, edited by B. Carter and J. B. Hartle (NATO Advanced Study Institute, Series B: Physics, Vol. 156) (Plenum Press, New York, 1986), pp. 229-235.