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# EXACT NON-SPHERICAL RADIATING COLLAPSE

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We study the junction conditions for non-spherical collapsing radiating star consisting of a shearing fluid undergoing radial heat flow with outgoing radiation. Radiation of the system is described by plane symmetric version of Vaidya solution. Junction conditions which match the collapse solutions to an exterior Vaidya metric show that, at the boundary, the pressure is proportional to the magnitude of the heat flow vector. Physical quantities, analogous to spherical symmetry related to the local conservation of momentum and surface red-shift, are also obtained. Finally, exact gravitational collapse solutions, for both shear and shear-free case, have been obtained by integrating a field equation.

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## 1. Introduction

Gravitational collapse is one of the most thorny and important problem in classical general relativity. It has many interesting applications in astrophysics where the formation of compact stellar objects such as white dwarf and neutron star are usually preceded by a period of radiative collapse. In the framework of Einsteins general theory of relativity the study of gravitational collapse requires to solve the Einstein equations for the collapsing fluid with a realistic equation of state and transport properties. It is also necessary to describe adequately the geometry of interior and exterior regions and to give conditions which allow matching of these regions.

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The pioneering work on gravitational collapse appeared in the famous paper of Oppenheimer and Snyder [1] in which they studied collapse of dust with a static Schwarzschild exterior whereas the interior space-time is represented by Friedman like solution. This work established an early frontier in our understanding of the evolution of the late stages of gravitational collapse. Since that time, many authors have added to a more realistic treatment of the collapse. The case with static exterior was studied by Misner and Sharp [2], for a perfect fluid in the interior. Outgoing radiation of the collapsing body has been considered by Vaidya [3]. It then become possible to model the radiating star by matching them to exterior Vaidya space-time [4,5]. The inclusion of the dissipation in the source by allowing radial heat flow while the body undergoes radiating collapse has been advanced by Santos and collaborators [5,6,7,8,9,10].

These studies were restricted to spherically symmetric space-times. On the other-hand, non-spherical collapse not so well understood. However, non-spherical collapse could occur in real astrophysical situation, and it is also important for a better understanding of both cosmic censorship conjecture [11] and hoop conjecture [12]. In-fact, collapse of cylindrical system led to the formulation of the hoop conjecture [12]. The plane symmetric models has also received significant attention owing to its close resemblance with spherically symmetric one. Exterior solutions to plane symmetric Einstien's field equations were obtained by Taub [13], while plane symmetric version of Vaidya solution were given by Dutta [14], and Carlson and Safko [15].

In ref. [6] the junction conditions for the spherically symmetric collapse of isotropic fluid undergoing radial heat flow with outgoing unpolarized radiation has been studied. The main objective of this paper is to extend is this work of Santos[6] to plane symmetric solutions. The interior space-time $\mathcal{V}_I$ is modeled by shearing fluid undergoing radial heat flow with outgoing radiation in a plane symmetric space-time. The exterior space-time $\mathcal{V}_E$ is described by the plane symmetric version of Vaidya space-time [14,15], which represent a radial flow of unpolarized radiation. This is done in section 3. In this section we also derive the formula for the total luminosity perceived at infinity and for the surface red-shift, which are of particular interest since they are observable quantities. In section 2 we give the field equations which govern the plane symmetric collapse of a radiating star with outgoing radiation. Exact gravitational collapse solutions with shear and radial heat flow are obtained in Section 4. In section 5 we find exact solutions for the shear-free case. Finally, concluding remarks and a summary of the main result obtained in this work are presented in section 6.

We have used the units which fix the speed of light and gravitational constant via $8\pi G = c = 1$.

## 2. Field Equations in Plane Symmetric Space-time

Let us consider a plane symmetric distribution of fluid undergoing dissipation in the form of heat flow. While the dissipative fluid collapses, it produces the unpolarized radiation.

We consider a plane surface with its motion described by a time-like 3-surface $\Sigma$, which divides space-times into interior and exterior manifolds $\mathcal{V}_I$ and $\mathcal{V}_E$. The interior space-time $\mathcal{V}_I$ is described by most general plane symmetric metric, which in comoving coordinates reads:

$$ds^2 = -A(r,t)^2 dt^2 + B(r,t)^2 dr^2 + C(r,t)^2(dx^2 + dy^2). \tag{1}$$

The exterior space-time is described by plane symmetric Vaidya metric [14,15], which represents an outgoing unpolarized radiation,

$$ds^2 = \frac{2m(v)}{\mathbf{r}} dv^2 - 2dvd\mathbf{r} + \mathbf{r}^2(dx^2 + dy^2). \tag{2}$$

The arbitrary function $m(v)$, represents the mass at retarded time $v$ inside the boundary surface $\Sigma$. We assume that the source of Einstein field equations in the interior space-time is given by

$$G^-_{ab} = \kappa T_{ab} = \kappa[(\zeta + p)u_a u_b + p g_{ab} + q_a u_b + q_b u_a], \tag{3}$$

where $\zeta$ is the energy density of fluid, $p$ denotes the isotropic pressure, $u_a$ is 4-velocity, and $q_a$ is radial heat flux satisfying $q_a u^a = 0$. Since we utilize comoving coordinates, we shall have

$$u^a = \frac{1}{A}\delta^a_0, \qquad q^a = q\delta^a_1. \tag{4}$$

The line element (1), in our plane symmetric case, yields the following Einstein's field equations

$$G^-_{00} = -\left(\frac{A}{B}\right)^2\left[2\frac{C''}{C} + \left(\frac{C'}{C}\right)^2 - 2\frac{C'B'}{CB}\right] + \left(\frac{\dot{C}}{C}\right)^2 + 2\frac{\dot{C}\dot{B}}{CB} = \kappa\zeta A^2, \tag{5}$$

$$G^-_{11} = \left(\frac{C'}{C}\right)^2 + 2\frac{A'C'}{AC} - \left(\frac{B}{A}\right)^2\left[2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - 2\frac{\dot{A}\dot{C}}{AC}\right] = \kappa p B^2, \tag{6}$$

$$\begin{aligned} G^-_{22} = &\left(\frac{C}{B}\right)^2\left[\frac{A''}{A} + \frac{C''}{C} + \frac{A'C'}{AC} - \frac{C'B'}{CB} - \frac{A'B'}{AB}\right] - \left(\frac{C}{A}\right)^2 \\ &\times\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{B}}{CB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{B}}{AB}\right] = \kappa p C^2, \end{aligned} \tag{7}$$

$$G^{2-}_2 = G^{3-}_3, \tag{8}$$

$$G^-_{01} = 2\left[\frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{A'\dot{C}}{AC}\right] = \kappa q A B^2, \tag{9}$$

plus an equation due to the isotropy of pressure

$$\frac{1}{A^2}\left[\frac{\ddot{C}}{C}-\frac{\ddot{B}}{B}+\left(\frac{\dot{C}}{C}\right)^2+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{A}\dot{C}}{AC}\right]$$
$$+\frac{1}{B^2}\left[\frac{A''}{A}+\frac{C''}{C}-\left(\frac{C'}{C}\right)^2-\frac{A'B'}{AB}-\frac{B'C'}{BC}-\frac{A'C'}{AC}\right]=0, \tag{10}$$

where the dot and the prime stand respectively for differentiation with respect to $t$ and $r$.

## 3. Junction Conditions

To study the junction conditions, we follow the approach of Santos [6]. Hence we have to demand, when approaching $\Sigma$ in $\mathcal{V}_I$ and $\mathcal{V}_E$, that the metrics be

$$(ds^2_-)_\Sigma = (ds^2_+)_\Sigma = (ds^2)_\Sigma, \tag{11}$$

where the subscript $\Sigma$ means that the quantities are to be evaluated on $\Sigma$ and let $K^{\pm}_{ij}$ is extrinsic curvature to $\Sigma$, defined by

$$K^{\pm}_{ij} = -n^{\pm}_{\alpha}\frac{\partial^2\chi^{\alpha}_{\pm}}{\partial\xi^i\partial\xi^j} - n^{\pm}_{\alpha}\Gamma^{\alpha}_{\beta\gamma}\frac{\partial\chi^{\beta}_{\pm}}{\partial\xi^i}\frac{\partial\chi^{\gamma}_{\pm}}{\partial\xi^j}, \tag{12}$$

and where $\Gamma^{\alpha}_{\beta\gamma}$ are Christoffel symbols, $n^{\pm}_{\alpha}$ the unit normal vectors to $\Sigma$, $\chi^{\alpha}$ are the coordinates of the interior and exterior space-time and $\xi^i$ are the coordinates that defines $\Sigma$. The intrinsic metric on the hypersurface $r = r_\Sigma$ is given by

$$ds^2 = -d\tau^2 + \mathcal{R}^2(\tau)(dx^2 + dy^2), \tag{13}$$

with coordinates $\xi^a = (\tau,\ x,\ y)$.

We use comoving coordinates and consider that the interior of the space-time $\mathcal{V}_I$ is described by line element (1). In this coordinate the surface $\Sigma$, being the boundary of the matter distribution, will have the equation

$$f(r,t) = r - r_\Sigma = 0, \tag{14}$$

where $r_\Sigma$ is a constant. The vector with component $\partial f/\partial\chi^a_-$ is orthogonal to $\Sigma$. Consequently the unit normal vector takes the form

$$n^-_a = B(r_\Sigma, t)\delta^1_a. \tag{15}$$

From the junction condition (11) we obtain

$$\frac{dt}{d\tau} = \frac{1}{A(r_\Sigma, t)}, \tag{16}$$

$$C(r_\Sigma, t) = \mathcal{R}(\tau). \tag{17}$$

The non-vanishing components of extrinsic curvature $K^{-}_{ij}$ of $\Sigma$ can be calculated as in the spherical case (for details see Santos[6]) and the result is

$$K^{-}_{\tau\tau} = \left[-\frac{A'}{AB}\right]_{\Sigma} \tag{18}$$

$$K^{-}_{x\,x} = \left[\frac{C'C}{B}\right]_{\Sigma} \tag{19}$$

$$K^{-}_{y\,y} = K^{-}_{x\,x}. \tag{20}$$

The equation for the surface $\Sigma$ in $\mathcal{V}_E$ is

$$f(\mathbf{r}, v) = \mathbf{r} - \mathbf{r}_{\Sigma} = 0, \tag{21}$$

therefore

$$\frac{\partial f}{\partial \chi^{a}_{+}} = \left(-\frac{d\mathbf{r}_{\Sigma}}{dv}, 1, 0, ..., 0\right), \tag{22}$$

and unit normal to $\Sigma$ is

$$n^{+}_{a} = \left[-\frac{2m(v)}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv}\right]^{-1/2} \left(-\frac{d\mathbf{r}}{dv}, 1, 0, ..., 0\right). \tag{23}$$

The first junction condition (11) for the line element (2) and (13) yields the following relations

$$\mathbf{r}_{\Sigma} = \mathcal{R}(\tau), \tag{24}$$

$$\left(\frac{dv}{d\tau}\right)^{-2}_{\Sigma} = \left[-\frac{2m(v)}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv}\right]_{\Sigma}. \tag{25}$$

With the help of (25) we can rewrite the normal vector as

$$n^{+}_{a} = (-\dot{\mathbf{r}}, \dot{v}, 0, ..., 0). \tag{26}$$

The non-vanishing components of extrinsic curvature $K^{+}_{ij}$ of $\Sigma$ are given by

$$K^{+}_{\tau\tau} = \left[\frac{d^2 v}{d\tau^2}\left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right)\frac{m(v)}{\mathbf{r}^2}\right]_{\Sigma}, \tag{27}$$

$$K^{+}_{x\,x} = \left[\mathbf{r}\frac{d\mathbf{r}}{d\tau} - 2m(v)\left(\frac{dv}{d\tau}\right)\right]_{\Sigma}, \tag{28}$$

$$K^{+}_{y\,y} = K^{+}_{x\,x}. \tag{29}$$

From Eqs. (19) and (28) we have

$$\left[-\frac{2m(v)}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv}\right]_{\Sigma} = \left[\frac{CC'}{B}\right]_{\Sigma}. \tag{30}$$

With the help of Eqs. (16), (17) and (25), we can write Eq. (30) as

$$m(v) = \frac{\mathbf{r}}{2}\left[\frac{\dot{C}^2}{A^2} - \frac{{C'}^2}{B^2}\right], \tag{31}$$

which can be interpreted as the total energy entrapped within the surface $\Sigma$. This expression (31) is analogous to well known mass function, in spherically symmetry case, introduced by Cahill and McVittie [16]. From Eqs. (27) and (18), using (16), we have

$$\left[\frac{d^2v}{d\tau^2}\left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right)\frac{m(v)}{\mathbf{r}^2}\right]_\Sigma = -\left(\frac{A'}{AB}\right)_\Sigma . \tag{32}$$

Substituting Eqs. (16), (17) and (31) into (30), results in

$$\left(\frac{dv}{d\tau}\right)_\Sigma = \left[\frac{C'}{B} + \frac{\dot{C}}{A}\right]^{-1}_\Sigma . \tag{33}$$

Differentiating (33) with respect to $\tau$ and using Eqs. (31), we can cast Eq. (32) as

$$\frac{1}{B^2}\left[\frac{C'^2}{C^2} + 2\frac{A'C'}{AC}\right] - \frac{1}{A^2}\left[2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - 2\frac{\dot{A}\dot{C}}{AC}\right] = \frac{2}{AB}\left[\frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{A'\dot{C}}{AC}\right]. \tag{34}$$

Comparing (34) with (6) and (9), we can finally write

$$(p)_\Sigma = (qB)_\Sigma, \tag{35}$$

which is equivalent to result obtained by Santos [6] for the spherically symmetry case. Equation (35) shows that for a plane symmetric shearing distribution of a collapsing fluid, undergoing dissipation in the form of heat flow, the isotropic pressure on the surface of discontinuity $\Sigma$ can not be zero. Clearly, if the fluid stops dissipation, i.e., $q_\Sigma = 0$, the pressure will vanish at the boundary which implies the radiation can not exist and exterior space-time $\mathcal{V}_E$ is a Taub space-time [13].

Differentiating partially (31) with respect to $t$ and utilizing (6) and (9), leads to

$$\left(\frac{\partial m}{\partial t}\right)_\Sigma = \left[\frac{dm}{dv}\frac{dv}{d\tau}\left(\frac{dt}{d\tau}\right)^{-1}\right]_\Sigma = -\left[\frac{\mathbf{r}^2}{2}\left(\kappa p\dot{C} + \kappa q A C'\right)\right]_\Sigma . \tag{36}$$

On using (16), (17), (33) and (35), we obtain

$$\left[-\frac{2}{\mathbf{r}^2}\frac{dm}{dv}\left(\frac{dv}{d\tau}\right)^2\right]_\Sigma = [\kappa p]_\Sigma . \tag{37}$$

Therefore, the total luminosity for an observer at rest at infinity is

$$L_\infty = \lim_{r\to\infty}\frac{\kappa}{2}\mathbf{r}^2\epsilon = -\left(\frac{dm}{dv}\right)_\Sigma = \left[\frac{1}{2}\,\kappa\,\mathbf{r}^2\,p\left(\frac{\dot{C}}{A} + \frac{C'}{B}\right)^2\right]_\Sigma , \tag{38}$$

where $dm/dv \leq 0$ since $L_\infty > 0$. Let the observer with 4-velocity is considered to be on $\Sigma$, the radiation energy density that this observer measures on $\Sigma$ is

$$\epsilon_\Sigma = \frac{2}{\kappa}\left[-\frac{1}{\mathbf{r}^2}\left(\frac{dv}{d\tau}\right)^2\frac{dm}{dv}\right]_\Sigma . \tag{39}$$

Inspection of Eqs. (37) and (39), reveals that

$$\epsilon_\Sigma = p_\Sigma. \tag{40}$$

This result is also valid in the analogous study in spherical symmetric system.[17] Equation (37) expresses the local conservation of momentum if we consider the momentum of radiation flowing in $\mathcal{V}_E$ and the energy density as given by (39).

Defining luminosity observed on $\Sigma$ as

$$L_\Sigma = \frac{\kappa}{2}\,\mathbf{r}^2\,\epsilon_\Sigma. \tag{41}$$

The boundary red-shift can be used to determine the time of formation of the horizon. The boundary red-shift $z_\Sigma$ of the radiation emitted by a star is given by

$$1 + z_\Sigma = \left(\frac{dv}{d\tau}\right)_\Sigma, \tag{42}$$

and thus the total luminosity $L_\Sigma$ perceived by an observer on $\Sigma$ is related with $L_\infty$ by the formula

$$(1 + z_\Sigma)^2 = \frac{L_\Sigma}{L_\infty}. \tag{43}$$

The results are analogous to one obtained previously in spherical case [5,6,7,8].

## 4. Exact solutions with shear

To find solutions of field equations, we introduce the following new variables

$$\alpha = AC, \qquad \beta = \frac{B}{C}. \tag{44}$$

The rate of shear scalar can be expressed in new variable as

$$\sigma = \frac{1}{\sqrt{3}A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = \frac{1}{\sqrt{3}}\frac{\dot{C}}{\alpha}\left(\frac{\dot{\beta}}{\beta} - 1\right), \tag{45}$$

and the pressure isotropy equation Eq. (10) becomes

$$\alpha^{-2}\beta^4C^4\left[\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - 4\frac{\dot{\beta}\dot{C}}{\beta C} - \frac{\ddot{\beta}}{\beta}\right] + \frac{\alpha''}{\alpha} - \frac{\alpha'\beta'}{\alpha\beta} - 4\frac{\alpha C'}{\alpha C} + 2\left(\frac{C'}{C}\right)^2 = 0. \tag{46}$$

**Solution A.** We choose $\alpha$ and $f$ of the form

$$\alpha = f\dot{\beta}C^4, \qquad f = f(r), \tag{47}$$

and the Eq. (46), with the help of this, yields the ordinary differential equation

$$F'' + C_1F' + C_2F = 0, \tag{48}$$

where

$$C = F^2, \tag{49}$$

$$C_1 = \frac{f'}{f} + \frac{\dot{\beta}'}{\beta} - \frac{\beta'}{\beta}, \tag{50}$$

$$C_2 = \frac{f''}{f} + \frac{f'}{f}\left(2\frac{\dot{\beta}'}{\dot{\beta}} - \frac{\beta'}{\beta}\right) + \frac{\dot{\beta}''}{\dot{\beta}} - \frac{\dot{\beta}'}{\dot{\beta}}\frac{\beta'}{\beta}. \tag{51}$$

Hence properly choosing the functions $f(r)$ and $\beta(r,t)$ and, integrating Eq. (48) , one can generate collapse solutions with shear and heat flow.

i)If we choose $f = f_0 =$ const. and $\beta = \sqrt{8}\tau(t)$, where $\tau(t)$ arbitrary, then we find that $F'' = 0$, which leads to

$$F = rg(t) + h(t), \tag{52}$$

where $g(t)$ and $h(t)$ are arbitrary integration functions. It may be noted that, in the analogous spherically symmetric case, one gets solutions in terms of trigonometric function [?]

ii) Next we choose $f = f_0 =$ const. and $\beta = K_0/(t+r)$, $K_0$ is constant, then we find that

$$F = \mu(t)\left[2(t+r)^2\right]^{(-1+\sqrt{2})/(2\sqrt{2})} + \nu(t)\left[2(t+r)^2\right]^{(1+\sqrt{2})/(2\sqrt{2})}, \tag{53}$$

where $\mu(t)$ and $\nu(t)$ are arbitrary integration functions.

iii) If the functions are chosen such that $f = f_0 e^{-2r/r_0}$, $f_0$ and $r_0$ const., $\beta = (2/r_0)\cos\tau$, $\tau(t)$ arbitrary. In this case we obtain

$$F = \psi(t)e^{(2-\sqrt{2})r/(2r_0)} + \omega(t)e^{(2+\sqrt{2})r/(2r_0)}, \tag{54}$$

where $\psi(t)$ and $\omega(t)$ are arbitrary integration functions.

**Solution B.** If we choose the functions $\alpha$ and $\beta$ such that

$$\alpha = \alpha_0 \dot{C}C, \qquad \beta = \frac{\beta_0}{C^2}, \tag{55}$$

where $\alpha_0$ and $\beta_0$ are two arbitrary constants, then we find that the pressure isotropy equation is an exact $\partial/\partial t$ of $CC'' = j(r)$, which, for $j(r) = 0$, integrates to

$$C = h_1(t)r + h_2(t), \tag{56}$$

where $h_1(t)$ and $h_2(t)$ are arbitrary function of integration.

## 5. Shear-free Solutions

Let us now consider solutions of the Einsteins equations for the interior metric of the form [10]

$$A = A_0(r), \tag{57}$$

$$B = B_0(r)f(t), \tag{58}$$

$$C = rB_0(r)f(t), \tag{59}$$

where$f(t)$ is positive.Then it follows that $A_0$ , $B_0$ are solutions of the pressure isotropy equation

$$\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right)' - \left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right)^2 - \frac{1}{r}\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right) + 2\left(\frac{A_0'}{A_0}\right)^2 = \frac{1}{r^2}, \tag{60}$$

describing a static perfect fluid whose energy density $\zeta_0$ and pressure $p_0$ are given by

$$\kappa\zeta_0 = -\frac{1}{B_0^2}\left[2\frac{B_0''}{B_0} - \left(\frac{B_0'}{B_0}\right)^2 + \frac{4B_0'}{rB_0} + \frac{1}{r^2}\right], \tag{61}$$

$$\kappa p_0 = \frac{1}{B_0^2}\left[\left(\frac{B_0'}{B_0}\right)^2 + 2\frac{A_0'B_0'}{A_0B_0} + \frac{2}{r}\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right) + \frac{1}{r^2}\right], \tag{62}$$

This static perfect fluid solution matches with the exterior Taub space-time and in this case its pressure $p_0$ vanishes for some value of r. We suppose that this happens for $r = r_\Sigma$

$$(p_0)_\Sigma = 0. \tag{63}$$

With the help of above Eqs. (57) - (62), the Eqs. (5), (6) and (9) can be rewritten as

$$\kappa\zeta = \frac{1}{f^2}\left(\kappa\zeta_0 + \frac{3\dot{f}^2}{A_0^2}\right), \tag{64}$$

$$\kappa p = \frac{1}{f^2}\left[\kappa p_0 - \frac{1}{{A_0}^2}(2f\ddot{f} + \dot{f}^2)\right]. \tag{65}$$

$$\kappa q = -\frac{2{A_0}'\dot{f}}{{A_0}^2{B_0}^2 f^3}, \tag{66}$$

as $(p)_\Sigma = (qB)_\Sigma$ and $(p_0)_\Sigma = 0$ we find

$$2f\ddot{f} + \dot{f}^2 - 2a\dot{f} = 0, \tag{67}$$

where the constant

$$a = \left(\frac{{A_0}'}{B_0}\right)_\Sigma, \tag{68}$$

is positive because the static solution $(A_0, B_0)$ matches with the Taub exterior metric. The first integral of equation (67) is

$$\dot{f} = -2a\left(\frac{b}{\sqrt{f}} - 1\right). \tag{69}$$

By remembering that $(p)_\Sigma$ is non-negative and by using (63), (65) and (67) we arrive at the important conclusion that the only possible motion of the system is contraction $\dot{f(t)} \leq 0$. Furthermore, from equation (69) and because $f(t)$ is positive we have $0 \leq f(t) \leq b^2$. Now,the integration of equation (69) yields

$$t = \frac{1}{a}\left[\frac{1}{2}f + b\sqrt{f} + b^2 \ln\left(1 - \frac{\sqrt{f}}{b}\right)\right], \tag{70}$$

where the constant of integration which should enter in equation(70) has been eliminated by means of the transformation $t \to t +$ const. The solution (57), (58) and (70) represents a static perfect fluid at $t = -\infty$ where $f(t) = 1$ and then the fluid gradually starts evolving into a non-adiabatic radiating collapse. The total energy entrapped inside the surface $\Sigma$ is given by(31), which becomes by utilizing the solution (57), (58) and (69),

$$m(v) = \left[\frac{2a^2 r^3 {B_0}^3}{{A_0}^2}(1 - \sqrt{f})^2 + m_0 f\right]_\Sigma, \tag{71}$$

with Eqs. (57), (58) and (69) $L_\infty$ given by(43) becomes

$$L_\infty = 2a^2\left(\frac{{B_0}^2 r^2}{{A_0}^2}\right)_\Sigma \frac{(1-\sqrt{f})}{\sqrt{f}} \frac{1}{(1+z_\Sigma)^2}, \tag{72}$$

with $z_\Sigma$ given by

$$z_\Sigma = \left[\frac{B_0{}'}{B_0}r - \frac{2aB_0 r}{A_0}\frac{(1-\sqrt{f})}{\sqrt{f}} + 1\right]_\Sigma^{-1} - 1. \tag{73}$$

Eq. (72) shows that $L_\infty \to 0$ for $f \to 1$. We observe that the function $f(t)$ decreases monotonically from the value $b^2$ at $t = -\infty$ to zero at $t = 0$ where a physical singularity is reached. It follows that the collapse begins at $t = -\infty$ from a static geometry, described by the interior solution $(A_0, b^2 B_0)$, whose energy density and pressure are given by Eqs. (61) and (62) provided that the right hand members of these equations are multiplied by a factor $b^{-2}$. From now on, for convenience we absorb the factor $b^2$ into $B_0(r)$ by setting $b = 1$. In this way the initial energy density and pressure of the geometry are given by Eqs. (61) and (62), its initial mass by

$$m_0 = -\left(\frac{r^3 {B_0{}'}^2}{2B_0} + r^2 B_0{}' + \frac{rB_0}{2}\right)_\Sigma, \tag{74}$$

representing the energy inside $\Sigma$ at $t = -\infty$ and its initial radius by

$$\mathbf{r_0} = (rB_0)_\Sigma. \tag{75}$$

For $t = -\infty$ the exterior space-time becomes the Taub space-time and by considering the junction conditions we have

$$ds^2 = \frac{2m_0}{\hat{r}}dt^2 - \frac{\hat{r}}{2m_0}d\hat{r}^2 + \hat{r}^2(dx^2 + dy^2), \tag{76}$$

$$A_0{}' = -\frac{1}{2\hat{r}}\sqrt{\frac{2m_0}{\hat{r}}}, \tag{77}$$

$$B_0 = \sqrt{\frac{\hat{r}}{2m_0}}, \tag{78}$$

which allows us to express Eq. (68) in terms of the initial quantities $m_0$ and $\mathbf{r_0}$ by using Eq. (75),

$$a = -m_0/\hat{r}^2, \tag{79}$$

where

$$\hat{r} = \mathbf{r_0}. \tag{80}$$

To obtain explicit dependence between the retarded time $v$ and the time t we must integrate Eq. (33). For this we write it in the form

$$\frac{dv}{df} = \left(\frac{A_0{}^2}{rB_0}\right)_\Sigma \frac{1}{\dot{f}(\dot{f}+h)}, \tag{81}$$

where

$$h = \left[\frac{A_0}{rB_0{}^2}(rB_0)'\right]_\Sigma. \tag{82}$$

This constant with the help of Eqs. (62), (63), (68), (75) and (79) can also be written in the form

$$h = \frac{2m_0}{\hat{r}^2}, \tag{83}$$

which shows that h is positive, otherwise the radius of the static plane would be less than the Taub radius. The luminosity $L$ can be calculated from Eqs. (38), (65), (67), (75), (79) and (81)

$$L = m_0\dot{f}(h^{-1}\dot{f} + 1)^2, \tag{84}$$

and the mass $m$ from Eqs. (31), (69), (74), (79) and (82)

$$m = m_0[(1-\sqrt{f})^2 + f]. \tag{85}$$

The differentiation of the luminosity $L$ given by the Eq. (84) with respect to $\dot{f}$ becomes

$$\frac{dL}{d\dot{f}} = m_0(h^{-1}\dot{f} + 1)\left[3h^{-1}\dot{f} + 1\right]. \tag{86}$$

Hence from Eqs. (84) and (86) we can conclude that the luminosity starts increasing from $L_0 = 0$ at the initial time $\dot{f} = 0$ and attains its maximum at

$$\dot{f} = -\frac{2}{3}\frac{m_0}{\hat{r}^2}, \tag{87}$$

$$L_{max} = -\frac{8}{27}\frac{m_0^2}{\hat{r}^2}. \tag{88}$$

After its maximum, the luminosity decreases to $L_H = 0$ at

$$\dot{f} = -\frac{2m_0}{\hat{r}^2}. \tag{89}$$

The energy density $\zeta$ and the isotropic pressure $p$ have to increases towards the center of the fluid distribution in order to be physically reasonable. Differentiating Eqs. (64) and (65) with respect to $r$ and utilizing (67), (69) and ($b = 1$), we obtain

$$\kappa\zeta' = \kappa\frac{\zeta_0'}{f^2} - 24a^2\frac{A_0'}{A_0^3}\frac{(1-\sqrt{f})^2}{f^3}, \tag{90}$$

$$\kappa p' = \kappa\frac{p_0'}{f^2} - 8a^2\frac{A_0'}{A_0^3}\frac{(1-\sqrt{f})}{f^{5/2}}, \tag{91}$$

$$\kappa q = \frac{4aA_0'}{A_0^2B_0^2}\frac{(1-\sqrt{f})}{f^{7/2}}, \tag{92}$$

and since we need $q > 0$ then we must have$A_0' > 0$. Hence from Eqs.(90) and (91) if $\zeta_0' < 0$ and $p_0' < 0$ are satisfied for the static distribution, then the fluid evolves satisfying the physical conditions $\zeta' < 0$ and $p' < 0$. It has been proved [18] that the energy conditions are fulfilled if the conditions $\zeta - 3p \geq 0$ together with $\zeta' < 0$ and $p' < 0$ are satisfied. From Eqs. (64), (65), (67), (69) and ($b = 1$) we can write

$$\kappa\zeta - 3\kappa p = \frac{1}{f^2}\left(\kappa\zeta_0 - 3\kappa p_0 + \frac{12a^2}{A_0^2}F\right), \tag{93}$$

where

$$F(f) = \frac{(1-\sqrt{f})(1-2\sqrt{f})}{f}. \tag{94}$$

The function $F(f)$ starts at $F = 0$ when $\sqrt{f} = 1$ and decreases to a minimum $F = -1/4$ at $\sqrt{f} = 2/3$. The metric $A_0$ decreases to the origin of the distribution, $A_0' > 0$. Hence we can be assured that $\zeta - 3p \geq 0$ is satisfied by the fluid throughout collapse if the initial static configuration satisfies

$$\kappa\zeta_0 - 3\kappa p_0 \geq \frac{3a^2}{A_0^2(r=0)}. \tag{95}$$

**More Solutions:** To simplify the pressure isotropy Eq. (10) let us define

$$x = r^2, \qquad B = \frac{1}{F}, \qquad C = rB \tag{96}$$

and then we have [20]

$$A_{xx} + 2\frac{F_x}{F}A_x - \frac{A}{4x^2} - \frac{F_{xx}}{F}A = 0. \tag{97}$$

i) Bergmann solution [20]

Let us adopt simplifying assumption that the fluid trajectories are geodesics. This is equivalent to considering $A = A(t)$ which permits us to set interior coordinate system so that $A = 1$. Now the above pressure isotropy equation can be written as $F_{xx} + F/(4x^2) = 0$ which gives general solution

$$B = \frac{1}{r\left[\alpha(t) + \beta(t)\log r\right]}. \tag{98}$$

ii) Modak [21] solution

The choice $F_{xx} = 0$ leads to the solution

$$A = \gamma(t) r^{-1+\sqrt{2}} + \delta(t) r^{-1-\sqrt{2}}, \qquad B = \frac{\xi(t)}{r^2}. \tag{99}$$

iii) The solution

$$A = \zeta(t) r^2, \qquad B = \frac{1}{\eta(t) r^{3-2\sqrt{2}} + \tau(t) r^{3+2\sqrt{2}}}, \tag{100}$$

is obtained under the assumption $A_{xx} = 0$.

iv) If

$$A(r,t) = 1 + \xi(t) r^2, \qquad B(r,t) = R(t), \tag{101}$$

this solution can be specialized by assuming $\xi(t) = \xi_0 =$ constant. The matching condition at the boundary surface yields a differential equation

$$2R\ddot{R} + \dot{R}^2 - m\dot{R} = n, \tag{102}$$

where

$$m = 4\xi_0 r_0, \qquad n = (4\xi_0 + \frac{1}{r_0^2})(1 + \xi_0 r_0^2), \tag{103}$$

a simple solution for $R(t) = Ct$, C being an arbitrary constant.where

$$C = \frac{m \pm \sqrt{m^2 + 4n}}{2}. \tag{104}$$

By suitable choices of constant parameters the density and the pressure are found to be positive, which are infinitely large as $t \to 0$ and decreases indefinitely as $t \to \infty$ [22].

v) With

$$A(r,t) = 1 + \xi_0 r^2, \qquad B(r,t) = \frac{R(t)}{(1 + \xi_0 r^2)^3}. \tag{105}$$

The differential equation and, the behavior of density and pressure remains same with

$$m = 4\xi_0 r(1 + \xi_0 r^2)^3, \qquad n = \frac{(1 + \xi_0 r^2)^6}{r^2}\left(1 - 6\xi_0 r^2 + 5{\xi_0}^2 r^4\right). \tag{106}$$

## 6. Concluding Remarks

To sum up, this extends the previous studies of junctions conditions for a collapsing radiating star with outgoing radiation in the spherically symmetry to a non-spherical case. Exact collapse solutions, for both shear and shear free case, have been generated. The physical properties of the solutions generated will be discussed elsewhere. The results obtained here may be a necessary ingredient for a study in planer collapse of a radiating star in plane symmetric space-time.

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## Appendix A. Physical quantities

**1.** Here we give the physical quantities associated with the metric (1). The components of shear scalar are

$$\begin{aligned}\sigma^r_r &= \frac{2}{3A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\\ \sigma^x_x &= -\frac{1}{3A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = \sigma^y_y \qquad \text{(A.1)}\end{aligned}$$

and shear scalar

$$\sigma = \frac{\sqrt{6}}{3A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) \qquad \text{(A.2)}$$

**2.** Expansion

$$\Theta[u] = \frac{1}{A}\left(\frac{\dot{B}}{B} + 2\frac{\dot{C}}{C}\right) \qquad \text{(A.3)}$$

**3.** The components of Electric and Magnetic components of Weyl tensor are

$$\begin{aligned}E_{rr} &= \frac{1}{3}\Big[\frac{A''}{A} - (\frac{C'}{C})' - \frac{C'}{C}\left(\frac{A'}{A} - \frac{B'}{B}\right)\\ &\quad -\frac{A'B'}{AB} - \frac{B^2}{A^2}\left(\frac{\ddot{B}}{B} - (\frac{\ddot{C}}{C}) + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) - \frac{\dot{A}\dot{B}}{AB}\right)\Big]\\ E_{xx} &= -\frac{1}{2}\frac{C^2}{B^2}[E_{rr}] = E_{yy}\\ H_{ab} &= \text{All components are zero.} \qquad \text{(A.4)}\end{aligned}$$

**4.** The Raychaudhary equation reads

$$\Big[\frac{1}{A^2}\left(\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-2\frac{\dot{A}\dot{C}}{AC}+2\frac{\ddot{C}}{C}\right)-\frac{1}{B^2}\left(\frac{A''}{A}-\frac{A'B'}{AB}+2\frac{A'C'}{AC}\right)$$
$$=\frac{1}{A^2}\left(\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-2\frac{\dot{A}\dot{C}}{AC}+2\frac{\ddot{C}}{C}+\frac{8}{3}\frac{\dot{C}^2}{C^2}+\frac{8}{3}\frac{\dot{B}\dot{C}}{BC}+\frac{2}{3}\frac{\dot{B}^2}{B^2}\right)$$
$$-\frac{1}{B^2}\left(\frac{A''}{A}-\frac{A'B'}{AB}+2\frac{A'C'}{AC}\right)\Big] \tag{A.5}$$

**5.** The Kretschmann scalar is

$$K=4\Big[\frac{1}{A^4}\big[2\frac{\dot{C}^2}{C^2}\left(\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}+1\right)-\frac{\dot{C}^4}{C^4}+\frac{\ddot{B}^2}{B^2}+\frac{\dot{A}^2\dot{B}^2}{A^2B^2}-2\frac{\dot{A}}{A}\left(\frac{\dot{B}\ddot{B}}{B^2}+2\frac{\dot{C}\ddot{C}}{C^2}\right)\big]$$
$$+\frac{1}{B^4}\big[2\frac{C'^2}{C^2}\left(\frac{A'^2}{A^2}+\frac{B'^2}{B^2}+\frac{C'^2}{C^2}\right)-\frac{C'^4}{C^4}+\frac{A''^2}{A^2}+\frac{A'^2B'^2}{A^2B^2}+2\frac{C''^2}{C^2}-2\frac{B'}{B}\left(\frac{A'A''}{A^2}+2\frac{C'C''}{C^2}\right)\big]$$
$$+\frac{1}{A^2B^2}\big[2\frac{\dot{B}}{B}\big[\frac{\dot{A}A''}{A^2}-4\frac{\dot{C}A'C'}{AC^2}-2\frac{\dot{B}C'^2}{BC^2}-\frac{\dot{A}A'B'}{A^2B}+2\frac{\dot{C}B'C'}{C^2B}+4\frac{C'\dot{C}'}{C^2}-2\frac{\dot{C}C''}{C^2}\big]$$
$$+2\frac{A'}{A}\left(2\frac{\dot{A}\dot{C}C'}{AC^2}-\frac{\dot{C}^2A'}{C^2A}+\frac{\ddot{B}B'}{B^2}-2\frac{\ddot{C}C'}{C^2}+4\frac{\dot{C}'\dot{C}}{C^2}\right)-2\frac{A''B''}{AB}-4\frac{\dot{C}'^2}{C^2}-2\frac{\dot{C}^2C'^2}{C^4}\big]\Big] \tag{A.6}$$

**6.** The Ricci scalar is

$$R=\frac{2}{A^2}\left(\frac{\ddot{B}}{B}+2\frac{\ddot{C}}{C}-\frac{\dot{A}\dot{B}}{AB}-2\frac{\dot{A}\dot{C}}{AC}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}^2}{C^2}\right)$$
$$-\frac{2}{B^2}\left(\frac{A''}{A}+2\frac{C''}{C}-\frac{A'B'}{AB}+2\frac{A'C'}{AC}-2\frac{B'C'}{BC}+\frac{C'^2}{C^2}\right) \tag{A.7}$$

## References


1. J.R. Oppenhiemer and H. Snyder, *Phys. Rev.* **56**, 455 (1939).
2. C.W. Misner and D. Sharp, *Phys. Rev.* **136**, b571 (1964).
3. P.C. Vaidya, *Proc Indian Acad Sci* **A33**, 264 (1951).
4. K. Lake, *Phys. Rev. D* **26**, 518 (1982).
5. N.O. Santos, *Phys. Lett.* **A106**, 296 (1984).
6. N.O. Santos, *Mon. Not. R. Astr. Soc.* **216**, 403 (1985).
7. A.K.G. de Oliveira, N.O. Santos and C.A. Kolassis, *Mon. Not. R. Astr. Soc.* **216**, 1001 (1985).
8. A.K.G. de Oliveira, J.A. de F. Pacheco and N. O. Santos, *Mon. Not. R. Astr. Soc.* **220**, 405 (1986).
9. A.K.G. de Oliveira and N. O. Santos, *Astrophys. J.* **312**, 640 (1987).
10. A.K.G. de Oliveira, C.A. Kolassis and N.O. Santos *Mon. Not. R. Astr. Soc.* **231**, 1011 (1988).
11. R. Penrose, *Riv. del Nuovo Cim.* **1**, 252 (1969); in *General Relativity, an Einstein Centenary Volume*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).

12. K. S. Thorne, in *Magic without magic*, ed. J. R. Klander, (Freeman, San Francisco, 1972).
13. A. Taub, *Phys. Rev.* **103**, 454 (1956).
14. B. K. Dutta, *Nuovo. Cimento* **A 47**, 568 (1967);
15. G. T. Carlson and J. L. Safko, *J. Math. Phys.* **19**, 1617 (1978).
16. M.E. Cahill and G.C. McVittie, *J. Math. Phys.* **11**, 1382 (1970).
17. N. A. Tomimura and F. C. P. Nunes, *Astrophys. Space Sci.***199**, 215 (1993).
18. C. A. Kolassis, N. O. Santos and D. Tsoubelis, *Classical Quantum Gravity* (1987).
19. E. N. Glass, *Gen. Rel. Grav.* **21**, 733 (1989).
20. O. Bergmann, *Phys. Lett.* **A82**, 383 (1981).
21. B. Modak, *J. Astrophys. Astron.* **5**, 317 (1984).
22. A. Banerjee, *Heat Flow in General Relativity - an IUCAA preprint (2003).*