Estimating Cosmological Parameters from CMBR: Systematic effects

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Abstract The fluctuations in the cosmic microwave background radiation depend on the cosmological parameters. We carry out Markov Chain Monte Carlo simulations to constrain the cosmological parameter space with the current CMB data for possible systematic effects[1] such as beam noncircularity in CMB measurements and incomplete sky coverage.

Keywords: methods:statistical, cosmic microwave background, cosmological parameters

1 Introduction

Almost five decades have passed since it was recognized that the relics of the hot Big Bang left the present Universe with a non-zero temperature, the Cosmic Microwave Background Radiation (CMBR) which are the seeds of structure formation. Ever since its discovery a galaxy of measurements have been made to probe the CMBR. The recent WMAP (Wilkinson Microwave Anisotropy Probe) with its much refined instruments gave a great deal of insight into the CMBR and thus our Universe. However, it is of utmost importance that experiments to measure the anisotropy of the CMBR should look carefully at the sources of error in observations and measurements. Even after foreground removal there are other sources of errors which can be both statistical such as photon noise and cosmic variance which is caused by incomplete sky coverage and systematic errors such as detector noise, beam uncertainty, fluctuations in Earth's atmospheric emission. Even with satellites, full sky measurements cannot be made as the Galactic plane must be discarded. For instance, in case of WMAP, only$\sim$ 85% of the sky could be utilized for CMB power spectrum estimation although a full sky measurement was made. Moreover, even after a sky map is made, uncertainty in both calibration and 'beam' [1] response cause angular dependent errors in the power spectrum, C_l thereby affecting the cosmological parameters. Therefore, any error in CMBR measurements will have direct consequences in constraining the cosmological parameters.

So far the noncircularity of beam has been sparingly addressed (e.g. [2], [3], [4] and references therein). In this work we consider the error in measurement which comes about due to noncircularity of the experimental beam[2], which can be a major issue in searches for fine-scale anisotropy. This issue is of major

[1] Although incomplete sky coverage is not a systematic error but we have taken it into account for the sake of completeness since it puts further limitations on CMB measurements.

[2] Here only non-rotating beams have been considered.

concern, in particular, for small scale measurements. Another limitation is the incomplete sky coverage which primarily affects the large scales. We follow the Bayesian approach using *Markov Chain Monte Carlo* (MCMC) technique to estimate the cosmological parameters for a beam-corrected WMAP data using the MCMC code *COSMOMC* [5]. We have also carried out an analysis to see how the cosmological parameters are affected when only a fraction of the sky is used (as is the case for many small-scale and balloon-borne experiments). In the sections to follow we briefly explain the systematic errors dealt with in this work, namely, the noncircularity of the experimental beam and incomplete sky coverage. This is followed by the methodology for carrying out the parameter estimation. Later we discuss the results of our parameter estimation and give some indicative conclusions.

2 Systematics of the Experiment

In this section we briefly describe the uncertainty caused in CMB measurements due to experimental 'beam' being noncircular which affects the small scales and incomplete sky coverage which mainly affects the large scales.

2.1 Noncircularity of Experimental Beam

In the past years, a number of experiments have produced reasonably accurate measurements of the power spectrum anisotropies in the cosmic microwave background radiation on a range of angular scales. The recent WMAP data indicates that the cosmic variance dominates to about $l = 350$ and beyond this errors are due to noise as well as calibration and/or beam uncertainty. Systematic errors must be removed for 'unbiased' estimation. Even though computationally costly but still these issues need to be tackled. Recently some faster methods have been developed to marginalize over such errors [3], [4].

The CMB temperature fluctuation $\overline{\Delta T}(\hat{\mathbf{n}})$ along a direction $\hat{\mathbf{n}} = (\theta, \phi)$ can be decomposed into spherical harmonics as follows

$$\overline{\Delta T}(\hat{\mathbf{n}}) \; = \; \sum_{l>o} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{\mathbf{n}}), \tag{1}$$

where the coefficients

$$a_{lm} = \int d\mathbf{\Omega} \, \Delta T(\hat{\mathbf{n}}) \, Y^*_{lm}(\hat{\mathbf{n}}) \tag{2}$$

express the power of $Y_{lm}(\hat{\mathbf{n}})$, the spherical harmonic functions evaluated in the direction $\hat{\mathbf{n}}$. The angular power spectrum C_l is defined in the usual way

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2 =< |a_{lm}|^2 > . \tag{3}$$

In real experiments the instrumental response (beam width, form of the beam, noise etc.) have to be taken into account. The temperature measured by the instrument in response to the signal $\Delta T(\hat{\mathbf{n}}')$ is then given by the convolution

$$\overline{\Delta T}^{\mathcal{R}}(\hat{\mathbf{n}}) = \int d\mathbf{\Omega} \; \mathfrak{B}(\hat{\mathbf{n}}, \hat{\mathbf{n}}') \; \Delta T(\hat{\mathbf{n}}') + \mathcal{N}, \tag{4}$$

where the experimental 'beam' response function $\mathfrak{B}(\hat{\mathbf{n}}, \hat{\mathbf{n}}')$ gives the sensitivity of the instrument at different angles around the pointing direction and $\mathcal{N}$ is the instrumental noise.

Usually the experimental beam response is assumed to be circularly symmetric about the pointing direction. However, any real beam response function has deviations from circular symmetry. Even the 'main lobe' can be a cause for error and not just the 'side lobe' response [1]. Experiments such as ARCHEOPS, MAXIMA, BOOMERANG and the recent WMAP all have significant noncircular beams. Fig. 1(a) shows one of the WMAP beams as an example of a distinct noncircular beam (notice the iso-contours in the figure). An elliptic gaussian beam profile pointed along the $\hat{z}$-axis (assuming, say, the north-pole) expressed in terms of the spherical polar coordinates (θ, ϕ) about the pointing direction is given by [5]

$$\mathfrak{B}(\hat{z}, \hat{\phi}) = \frac{1}{2\pi\sigma_1\sigma_2} \exp\left[-\frac{\theta^2}{2\sigma^2(\phi)}\right] \tag{5}$$

where the beam width $\sigma(\phi) = \left[\sigma_1^2/(1+\epsilon\sin^2(\phi)\right]^{1/2}$ and the noncircularity parameter $\epsilon = (\sigma_1^2/\sigma_2^2 - 1)$ are given in terms of σ_1 and σ_2, the gaussian widths along the semi-major and semi-minor axis, respectively. Here, the noncircularity parameter has been replaced by the eccentricity $e = \sqrt{1 - \sigma_2^2/\sigma_1^2}$ [3].

Given the observed temperature fluctuations $\overline{\Delta T}^{\mathcal{R}}(\hat{\mathbf{n}})$, the angular power spectrum of Eq.(3) gives rise to a biased $C_{l'}$ (also called *Pseudo*-C_l) and is given by [3]

$$\tilde{C}_l = \sum_{l'} A_{ll'}\, C_{l'} \tag{6}$$

where $A_{ll'}$ is the *bias* matrix which arises due to noncircular beam, instrumental noise, incomplete sky coverage. The unbiased C_l 's are then obtained as

$$\tilde{C}_l^{UB} = B_l^{-2}\,(\tilde{C}_l - C_l^{\mathcal{N}}) \tag{7}$$

where B_l^{-2} is the beam transform function. Fig. 1(b). shows the angular power spectrum obtained by using a elliptical gaussian beam corrected with eccentricity $e = 0.6$ and beam width $\sigma = 0.0016$ compared with the angular power spectrum with no beam correction. We have carried out the MCMC analysis for such a beam-corrected WMAP data.

2.2 Incomplete Sky Coverage

In general, incomplete sky coverage mitigates information about the power spectrum by increasing the sample variance [8] and by obscuring features in the power spectrum (limited by the so-called resolution width $\Delta l \approx 1/\Delta\theta$, where Δl is the uncertainty in cosmic variance and θ is the smallest angular survey in radians, respectively [9]). Since the CMB data available for parameter estimation would certainly depend on the sky coverage we have been motivated to test the effect of a finite sky coverage along with the noncircularity of the beam. In general, the fraction of the sky covered within a sky area of angle ϑ is given by $f_{sky} = sin^2\vartheta/2$. We have carried out our study for 85%, 60% and 35% of the sky covered and estimated the cosmological parameters for each case.

3 Methodology of Parameter Estimation

Our approach to Bayesian parameter estimation makes use of *Markov chain Monte Carlo* (MCMC) sampling to explore the posterior distribution of the cosmological parameters. In general, for any given model $\mathcal{M}$, samples are drawn from the (unnormalized) posterior distribution given by the product of the likelihood and prior, i.e. $\mathcal{P}(d|\mathcal{Z},\mathcal{M})\ \mathcal{P}(\mathcal{Z}|\mathcal{M})$, where '*d*' denotes the data under analysis and $\mathcal{Z}$ denotes the parameters defining the model. The likelihood function is evaluated in the same manner as discussed in [5] using *COSMOMC*. The joint prior is simply the product of the individual priors. The model being considered here is the hot Big Bang ΛCDM model with primordial fluctuations which are assumed to be adiabatic and gaussian random fields. All parameters used here have a flat prior. We use the WMAP TT data (temperature mode only) which has been corrected for noncircularity. The cosmological parameters considered in the study are $\Omega_b h^2$ and $\Omega_c h^2$, the physical baryon and CDM densities relative to the critical density, respectively, $h = H_o/(100\ \mathrm{km\ s^{-1}\ Mpc^{-1}})$ the Hubble parameter, Ω_Λ the ratio of the critical density in the form of dark energy, the optical depth τ and the redshift at reionization z_{re}. Apart from these parameters initial perturbations during inflation are characterized by the amplitudes (A_S, A_T) and spectral indices of scalar and tensor perturbations (n_s, n_t).

4 Parameter estimates with Beam-corrected WMAP data and Incomplete Sky coverage

It can be noted from Fig. 1(b) that the power spectrum is affected by putting beam-correction into the WMAP data. This obviously suggests a likely change in certain cosmological parameters and possibly in their error bars. We have noted, in particular, changes in parameters, namely, $\Omega_b h^2$, $\Omega_c h^2$, Ω_Λ, τ and z_{re}. The baryonic content in the Universe, Ω_b is directly related to the height of the peaks of the power spectrum. We find a slight increase in $\Omega_b h^2$ and $\Omega_c h^2$ which is more pronounced for a scale invariant power spectrum ($n_s = 1$). The values of τ and the related z_{re} indicate the ionisation history of the Universe. If reionization took place sufficiently long ago then τ, of ionized gas would be greater than 1 and subsequent disappearance of intrinsic fluctuations. If later, then τ would be less and the fluctuations would be preserved[3]. We have done the analysis with beam-corrected WMAP TT data only considering the change in diagonal elements of the covariance matrix. In general, with TT data a lower value of τ leads to increase of power at large l. In fact, we notice that beam correction does lead to reduction in τ. We find that the values of τ and z_{re} decrease by $\sim$ 5-7% within the WMAP error bars (Fig. 2). This reduction in τ is more prominent with decrease in the scalar spectral index, n_s (Reduction in τ=6%,7% for n_s=0.98,0.95, respectively). Since the reionization is affected to a great extent by the TE and polarization data so we expect this decrease to be more pronounced when including these data as well. In fact, we notice such a trend when we include the WMAP TE data along with our beam-corrected

[3]The high values of $\tau = 0.17 \pm 0.04$ and $z_{re} = 20^{+10}_{-9}$ for WMAP includes both WMAP TT+TE data along with other CMB datasets [10].

TT data. We also notice a slight change in the dark energy content, Ω_Λ with the beam-corrected WMAP TT data (Ω_Λ decreases slightly but again increases slightly when WMAP TE data is included). With the projected WMAP two-year data and further six-year data release, WMAP data will be able to constrain the cosmological parameters with better accuracy (e.g. the two-year projected accuracy for τ is of the order of 13% as compared with 23% now). Hence, the parameter estimates mentioned here are most likely to get refined.

Recall that WMAP's effective sky coverage is $\sim$ 85% even for a full-sky measurement. Keeping this in mind we have carried out our study when 85%, 60% and 35% of the sky is covered and estimated the cosmological parameters for each case. We find that as the sky coverage decreases progressively from 85% to 35% the error bars on the cosmological parameters become large (In Fig. 3 notice the change in $1\sigma, 2\sigma$ contours as the sky coverage decreases). This is expected since the cosmic variance goes up by a factor $\sim 1/A$, 'A' being the area covered in the experiment [8]. Although we assume that the cosmic variance is large only for large scales and small ($\lesssim 1^o$) for small scales but this is true only for a full-sky measurement [8]. Along with different sky coverages, we have used the beam-corrected WMAP data in our study wherein the power spectrum gets affected mainly at small scales and we notice that error bars do get enhanced. Therefore, we have been able to demonstrate and confirm through this study that sample variance[4] can be a major source of uncertainty even for small scales and always puts limitation on the utility of the measurements.

5 Discussion and Conclusion

We would like to mention that the results discussed here are indicative of possible trends in the cosmological parameters due to beam uncertainty and incomplete sky coverage. As the WMAP mission progresses further the noise will integrate down and the knowledge of beams will continue to improve. With the future six-year WMAP data release, the epoch of reionization and other parameters will be much better constrained. Therefore, beam correction to the future WMAP data will give better estimates to cosmological parameters with reduced error bars. The temperature-polarization (TE) and polarization (EE) data will similarly require correction due to such effects. The WMAP data is limited by cosmic variance up to $l = 350$ and $S/N > 1$ up to $l \sim 650$. In addition, the WMAP data has been extended beyond $l = 700$ by using other datasets (CBI, ACBAR) which we have not included in our analysis as we wanted to see the effect of the systematics only on the current WMAP data. We would definitely expect the parameter estimates to vary further due to the above as well as for effects not taken into consideration here, for instance, polarization data, scanning and chopping strategy, scan pattern, rotating beams etc. Moreover, the effect of noncircularity of beam will become more relevant for experiments with high sensitivity and angular resolution such as PLANCK since beam uncertainty primarily affects the large multipoles. We would like to emphasize that beam uncertainty can certainly make significant difference to cosmological parameter estimation and hence, for all future missions which probe CMBR, this issue must be dealt with seriously.

[4] The relation between sample variance σ^2_{sam} and cosmic variance σ^2_{cos} can be taken as $\sigma^2_{sam} \backsimeq (4\pi/A)\,\sigma^2_{cos}$ [8].

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References

[1] In radio astronomy we make use of the word 'beam' to refer to the response of a radio telescope, even when it is receiving (and not transmitting) radiation from a limited solid angle of the sky. To a first approximation, that solid angle is determined by diffraction theory. It is thus usual to refer to the central maximum of the diffraction pattern as the 'main beam' and the secondary diffraction maxima as 'side lobes'. Further details can be found at [11].

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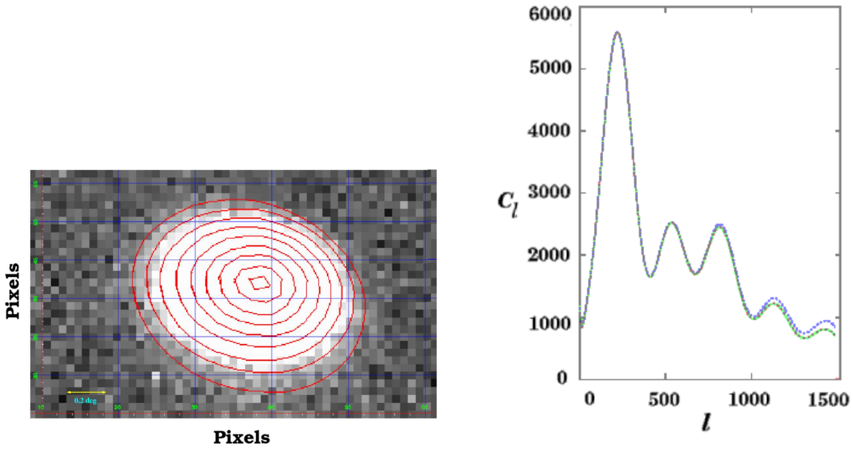

Figure 1: (a) The figure on the left shows the WMAP Q1 Beam map overlaid with IRAF fitted ellipses over iso-intensity contours. (b) The figure on the right shows the $C_l s$ for WMAP best fit and Beam-corrected WMAP data. The solid bottom line indicates the WMAP best fit while the dotted top line indicates the Beam-corrected power spectrum, the effect being more pronounced at large l (adapted from [3]).

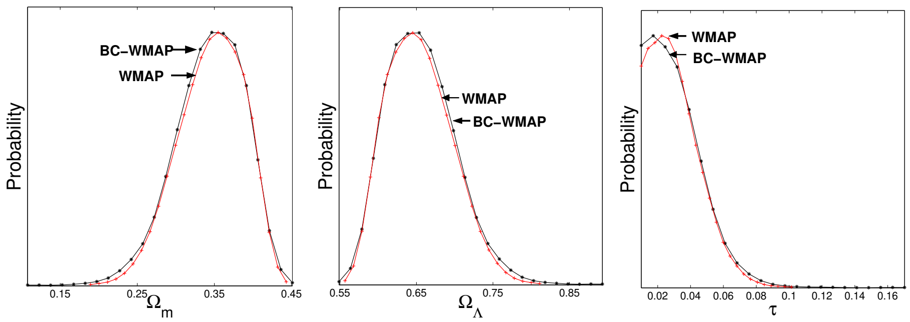

Figure 2: The plots show fully marginalized posterior likelihoods for the cosmological parameters ($\Omega_m, \Omega_\Lambda, \tau$ from left to right) which tend to show a change when beam correction is incorporated into the parameter estimation ('BC-WMAP' refers to Beam-corrected WMAP and 'WMAP' refers to WMAP only).

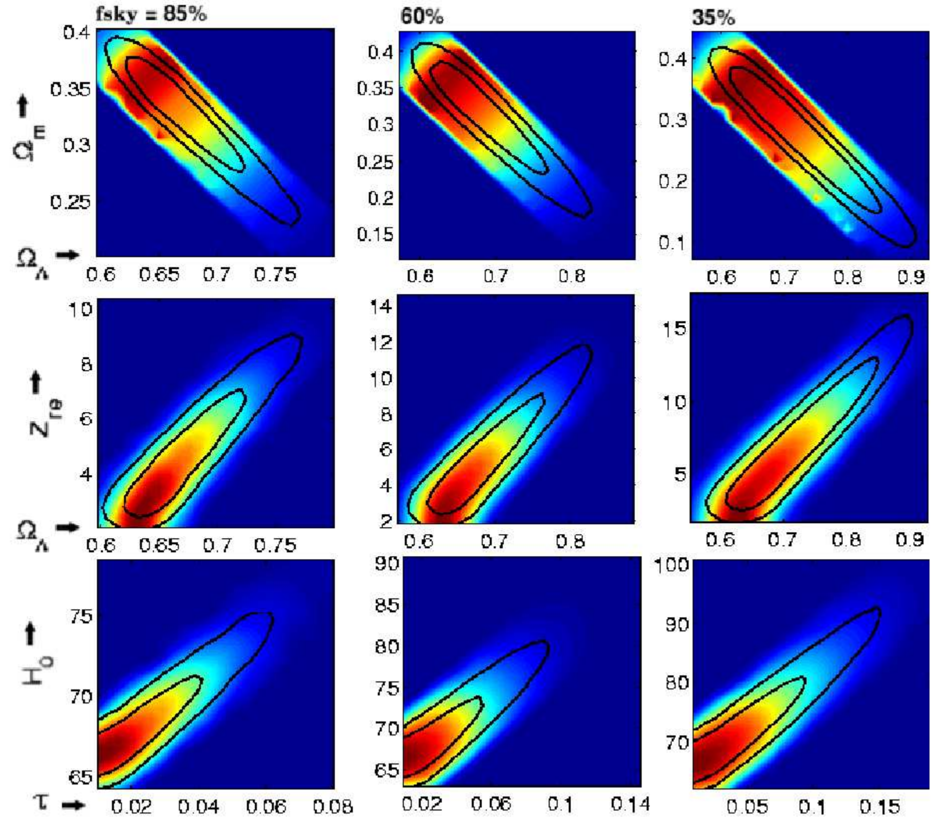

Figure 3: The plots show posterior constraints for the cosmological parameters with 85%, 60% and 35% sky coverage. The error bars progressively increase for reduced sky coverage. (The contours are 68% and 95% confidence limits from the marginalized distribution while the shading shows mean likelihood of the samples).