Non-spherical gravitational collapse of strange quark matter

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Abstract

We study the non-spherical gravitational collapse of the strange quark null fluid. The interesting feature which emerges is that the non-spherical collapse of charged strange quark matter leads to a naked singularity whereas the gravitational collapse of neutral quark matter proceeds to form a black hole. In the present brief report we extend the earlier work of T.Harko and K.S.Cheng to the non-spherical case.

Key words. Gravitational collapse; naked singularity; black hole; strange quark matter.

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1. Introduction

In past thirty years, there have been extensive studies on gravitational collapse in order to investigate the nature of the singularities. The study of gravitational collapse of spherically symmetric spacetimes led to many examples of naked singularities [1-10]. Most of these papers concentrate on the aim to find light rays emanated from the central singularity and escaping the Schwarzschildian trapped surface at least locally. In order to study the nature of the singularity, global behaviour of radial null geodesics must be studied in full generality.

For more than thirty years, cosmic censorship hypothesis (CCH) has been one of the most active areas of research in general relativity. CCH was proposed by R.Penrose[11], which states that singularities formed in gravitational collapse of physically reasonable matter can not be observed. There are two versions of this hypothesis. The weak CCH states that all singularities formed in gravitational collapse are hidden behind the event horizon of the gravity and are invisible to the distant observer from infinity. On the other hand, the strong CCH asserts that no singularities are visible. Many researchers have attempted to give precise reformulation to this hypothesis, but neither proof nor mathematical formulation for this hypothesis is available so far[12,14]. Much work has been done on CCH in spherically symmetric spacetimes. Almost all research papers which have been published on this issue till date, are limited to the collapse in spherically symmetric spacetimes. Of course, spherical symmetry is a characteristic feature of many solutions of Einstein's field equations of general relativity, but to prove (or disprove) the CCH, it is essential to study the gravitational collapse in non-spherical case as well. The purpose of this brief report is to provide a brief guide to the issue of naked singularities in non-spherically symmetric spacetimes.

One of the hot topics in astrophysics of late has been the possibility of the discovery of strange stars (stars composed of u, d, and s quark matter). A quark star or strange star is a hypothetical type of star composed of quark matter or strange matter. It is theorized that when the neutron degenerate matter which makes up a neutron star is put under sufficient pressure due to the star's gravity, the individual neutrons break down into their constituent quarks - up quarks and down quarks. Some of these quarks may then become strange quarks and form strange matter. The star which then becomes is known as a *quark star* or *strange star*. The broader meaning of strange matter is just quark matter that contains three flavors of quarks

- up, down and strange. In this definition, there is a critical pressure and an associated critical density; and when nuclear matter is compressed beyond this density, the protons and neutrons dissociate into quarks, yielding strange matter. Many research papers on the strange quark matter have appeared so far, explaining the formation and properties of strange stars [15-19]. The strange quark matter is characterized by the equation of state

$$P = \frac{\rho - 4B}{3}, \tag{1}$$

where B is the difference between the energy density of the perturbative and non-perturbative QCD vacuum, known as the *bag constant*. ρ and P are respectively, the energy density and thermodynamic pressure of the quark matter.The typical value of the bag constant is of the order of $10^{15} g/cm^3$, while the energy density $\rho \approx 5 \times 10^{15} g/cm^3$.(For details, see Ref.[20]). From the equation of state (1) it is obvious that the strange quark matter will always satisfy the energy conditions $\rho \geq P \geq 0$.

T. Harko and K.S. Cheng[20] have studied the gravitational collapse of strange matter and analyzed the condition for formation of a naked singularity in the spherically symmetric Vaidya like spacetime. It has been shown that depending on the initial distribution of density and velocity and on the constitutive nature of the collapsing matter, either a black hole or a naked singularity is formed.

The purpose of this brief report is to see how the earlier results that were investigated in [20] get modified for non-spherically symmetric(plane symmetric and cylindrically symmetric) spacetimes.

The plan of this brief report is as follows: In Sec.2, we obtain the general solution for strange quark matter, with the equation of state $P = (\rho - 4B)/3$, in non-spherically symmetric spacetimes. In Sec.3, we discuss the nature of the singularity by analyzing the equations of the outgoing radial null geodesics. We conclude the report in Sec.4.

2. Non-spherical collapse of strange quark matter

Following the work given in [20,22,23] the line element describing the radial collapse of charged strange quark fluid in toroidal,cylindrical or planar spacetime can be written as

$$ds^2 = -\left(\alpha^2 r^2 - \frac{qm(u,r)}{r}\right) du^2 + 2dudr + r^2 \left(d\theta^2 + d\phi^2\right) \tag{2}$$

Here u is an advanced Eddington time coordinate, and $\alpha = \sqrt{-\Lambda/3}$. r is the radial coordinate with $0 < r < \infty$, and $m(u,r)$ is the mass function giving gravitational mass inside the sphere of radius r. Coordinates θ, ϕ describe the two dimensional zero-curvature space generated by the the two-dimensional commutative Lie group G_2 of isometries[22]. Referring to [22,23], we write the topology of two-dimensional space:

Topology of toroidal model is $S \times S$, cylindrical model has topology $R \times S$, while planar symmetrical model has $R \times R$.
Ranges for θ and ϕ in the models are:

$$
\begin{array}{lll}
\text{(i) Toroidal:} & 0 \leq \theta < 2\pi \ , & 0 \leq \phi < 2\pi. \\
\text{(ii) Cylindrical:} & -\infty < \theta < \infty \ , & 0 \leq \phi < 2\pi. \\
\text{(iii) Planar:} & -\infty < \theta < \infty \ , & -\infty < \phi < \infty.
\end{array}
$$

Depending upon the topology of the two-dimensional space, parameter q has different values. For torous model, $m(u,r)$ is mass and $q = 2/\pi$. For the cylindrical case $m(u,r)$ is mass per unit length and $q = 4/\alpha$ and for planar symmetrical model $m(u,r)$ is mass per unit area and $q = 2/\alpha^2$. The values of q are taken from Arnowitt-Deser-Misner (ADM) masses of the corresponding black holes[23]. The energy momentum tensor for the solution (2) can be written in the form[20,24]

$$T_{\mu\nu} = T^{(n)}_{\mu\nu} + T^{(m)}_{\mu\nu} + E_{\mu\nu}, \tag{3}$$

where

$$T^{(n)}_{\mu\nu} = \sigma(u,r) l_\mu l_\nu, \tag{4}$$

is the component of the matter field that moves along the null hypersurface $u = const.$,

$$T^{(m)}_{\mu\nu} = (\rho + P)\left(l_\mu n_\nu + l_\nu n_\mu\right) + P g_{\mu\nu}, \tag{5}$$

represents the energy momentum tensor of the strange quark matter, and

$$E_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta}\right), \tag{6}$$

is the electromagnetic contribution. We have considered the null vectors l_μ, n_ν such that

$$
\begin{gathered}
l_\mu = \delta^0_\mu, \qquad n_\mu = \frac{1}{2}\left(\alpha^2 r^2 - \frac{q m(u,r)}{r}\right)\delta^0_\mu - \delta^1_\mu, \\
l_\lambda l^\lambda = n_\lambda n^\lambda = 0, \qquad l_\lambda n^\lambda = -1.
\end{gathered} \tag{7}
$$

The electromagnetic tensor $F_{\mu\nu}$ obeys the Maxwell equations[20]

$$\frac{\partial F_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial F_{\lambda\mu}}{\partial x^{\nu}} + \frac{\partial F_{\nu\lambda}}{\partial x^{\mu}} = 0, \tag{8}$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^{\mu}} \left(\sqrt{-g}\; F^{\mu\nu}\right) = -4\pi J^{\mu}. \tag{9}$$

Without loss of generality the electromagnetic vector potential can be chosen as

$$A_{\mu} = \frac{e(u)}{r}\; \delta^{(\mu)}_{(\mu)}, \tag{10}$$

where $e(u)$ is an arbitrary integrable function. From Eqs.(8) and (9) it follows that the only non-vanishing components of $F_{\mu\nu}$ are

$$F_{ru} = -F_{ur} = \frac{e(u)}{r^2},$$

and hence

$$E^{\nu}_{\mu} = \frac{e^2(u)}{r^4}\; diag(-1, 1, -1, 1). \tag{11}$$

Gravitational field equations for the energy-momentum tensor (3) can be written as

$$\sigma = \frac{q\dot{m}}{8\pi r^2}, \tag{12}$$

$$\rho = \frac{qm' - 3\alpha^2 r^2}{8\pi r^2} - \frac{q^2 e^2(u)}{8\pi r^4}, \tag{13}$$

$$P = \frac{6\alpha^2 r - qm''}{16\pi r} - \frac{q^2 e^2(u)}{16\pi r^4}. \tag{14}$$

The energy conditions for the above type of fluids are given by [20,24]

i) The weak and strong energy conditions:

$$\sigma > 0, \quad \rho \geq 0, \quad P \geq 0. \tag{15}$$

ii) The dominant energy conditions:

$$\sigma > 0, \quad \rho \geq P \geq 0. \tag{16}$$

Combining the bag equation of state (1) with Eqs.(13) and (14), we get

$$3qm'' r^2 + 2qm' r = 64\pi B r^3 + 24\alpha^2 r^3 - \frac{q^2 e^2(u)}{r}. \tag{17}$$

Solving the above differential equation we obtain the following general solution

$$qm(u,r) = qg(u) + qh(u)r^{1/3} + Ar^3 - \frac{q^2e^2(u)}{4r}, \tag{18}$$

where $g(u)$ and $h(u)$ are two arbitrary functions of u and $A = (8\pi B/3) + \alpha^2$. Substituting the above mass function in Eq.(2), we obtain the following solution to the Einstein equations for collapsing strange quark matter in toroidal (cylindrical or planar) spacetime

$$ds^2 = -\left(\alpha^2r^2 - \frac{qg(u)}{r} - \frac{qh(u)}{r^{2/3}} - Ar^2 + \frac{q^2e^2(u)}{4r^2}\right)du^2 \\ +2dudr + r^2\left(d\theta^2 + d\phi^2\right). \tag{19}$$

Using the mass function (18) we obtain the following quantities

$$\sigma = \frac{1}{8\pi r^2}\left(q\dot{g}(u) + q\dot{h}(u)r^{1/3} - \frac{q^2e(u)\dot{e}(u)}{2r}\right). \tag{20}$$

$$\rho = \frac{1}{8\pi r^2}\left(\frac{1}{3}qh(u)r^{-2/3} + 8\pi Br^2 - \frac{3}{4r^2}q^2e^2(u)\right). \tag{21}$$

$$P = \frac{1}{16\pi r}\left(\frac{2}{9}qh(u)r^{-5/3} - 16\pi Br - \frac{q^2e^2(u)}{2r^3}\right). \tag{22}$$

We note that with suitable choices of $g(u)$ and $h(u)$, weak and strong energy conditions are satisfied. Due to the bag equation of state (1) we always have $\rho \geq P \geq 0$ and thus dominant energy conditions hold too.

3. Nature of the Singularity

To investigate the structure of the collapse we need to consider the radial null geodesics defined by $ds^2 = 0$ taking into account $\dot{\theta} = \dot{\phi} = 0$. Then Eq.(19) becomes

$$\frac{dr}{du} = \frac{1}{2}\left(\alpha^2r^2 - \frac{qg(u)}{r} - \frac{qh(u)}{r^{2/3}} - Ar^2 + \frac{q^2e^2(u)}{4r^2}\right). \tag{23}$$

In general, the above equation does not yield analytical solution. However if $qg(u) \propto u$, $\quad qh(u) \propto u^{2/3}$ and $q^2e^2(u) \propto u^2$, then this equation becomes homogeneous and can be solved in terms of elementary functions. In particular, let us choose

$$qg(u) = \lambda u, \quad qh(u) = \beta u^{2/3} \quad \text{and} \quad q^2e^2(u) = \delta^2u^2, \tag{24}$$

for some $\lambda > 0, \beta > 0$ and $\delta \geq 0$. Then Eq.(23) becomes

$$\frac{du}{dr} = \frac{2}{\left(\alpha^2 r^2 - \frac{\lambda u}{r} - \frac{\beta u^{2/3}}{r^{2/3}} - A r^2 + \frac{\delta^2 u^2}{4r^2}\right)}. \tag{25}$$

It can be observed that the above differential equation has singularity at $r = 0, u = 0$. To discuss the nature of this singularity we analyze the outgoing radial null geodesics terminating at the singularity in the past. We follow the technique described in Ref.[25]. Let

$$X_0 = \lim_{\substack{u \to 0 \\ r \to 0}} X = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{u}{r}. \tag{26}$$

Hence Eq.(25) can be written as

$$X_0 = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{du}{dr} = \frac{8}{-4\lambda X_0 - 4\beta X_0^{2/3} + \delta^2 X_0^2}, \tag{27}$$

i.e.

$$\delta^2 X_0^3 - 4\lambda X_0^2 - 4\beta X_0^{5/3} - 8 = 0. \tag{28}$$

This algebraic equation decides the nature of the singularity. If the above equation has a real and positive root, then there exist future directed radial null geodesics originating from $r = 0, \quad v = 0$. In this case the singularity will be naked. If Eq.(28) has no real and positive root, then the singularity will be covered and the collapse proceeds to form a black hole.
Setting $X_0 = y^3$ in Eq.(28), we obtain

$$\delta^2 y^9 - 4\lambda y^6 - 4\beta y^5 - 8 = 0. \tag{29}$$

To analyze the the nature of the root of Eq.(29), the following rule from the *theory of equations* may be useful: *Every equation of odd degree has at least one real root whose sign is opposite to that of its last term, the coefficient of the first term being positive* . Accordingly, Eq.(29) has at least one positive root.

In particular, if $\delta = 0.1, \lambda = 0.1$ and $\beta = 0.01$ then one of the roots of Eq.(29) is $y = 1.2916$. Back substitution gives $X_0 = 2.1545$. This positive root of Eq.(28), ensures that the singularity is naked.

In the above discussion we have considered the non-spherical gravitational collapse of charged strange quark matter. It would be interesting to see whether the gravitational collapse of uncharged (i.e. $q^2e^2(u) = 0$) strange quark matter leads to a naked singularity or not.

For the uncharged case, we put $q^2e^2(u) = 0$ into Eq.(18). Then Eq.(28) reduces to

$$4\lambda X_0^2 + 4\beta x_0^{5/3} + 8 = 0 \tag{30}$$

As all the coefficients in Eq.(30) are positive, we conclude, from the *theory of equations* that, there is no positive root to this equation. In other words, outgoing radial null geodesics having definite tangent at the singularity in the past are absent, hence the singularity arising in this case is not naked. Thus the gravitational collapse proceeds to form toroidal (cylindrical or planar) black holes.

4. Conclusion

The motivation for this work originated from the need for obtaining and studying the nature of the singularities in non-spherically symmetric spacetimes. In this paper we have studied the visibility of naked singularities, investigating the behavior of radial null geodesics in the non-spherical gravitational collapse of strange quark matter. It is found that charge plays a crucial role in the non-spherical gravitational collapse of strange quark matter. The interesting feature which emerges is that the non-spherical collapse of charged strange quark matter leads to a naked singularity whereas the gravitational collapse of neutral quark matter proceeds to form a black hole.

Thus non-spherical gravitational collapse of charged strange matter contradicts the CCH, whereas collapse of neutral strange matter respects it.

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