Nucleosynthesis and Stellar Evolution

Nuclear fusion rates are extremely sensitive to temperature. In order to understand why this is so, we need to examine how a fusion process operates: since the positively charged nuclei which need to fuse have to face a strong coulomb barrier (see fig. 1), the fusion process operates mainly by tunnelling. In the centre of mass frame, if the kinetic energy of the approaching particles is E, then the tunnelling probability

$$P_0(E) \propto E^{-1/2} \exp(-2\pi\eta)$$

where

$$\eta = \left(\frac{m}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar} \frac{1}{E^{1/2}} = \frac{\bar{\eta}}{E^{1/2}}$$

m being the reduced mass of the two nuclei and Z_1, Z_2 their charge numbers.

At a given temperature T, the probability of a particle having an energy E is given by the Maxwell-Boltzmann distribution

$$f(E) \propto E^{1/2} \exp(-E/kT)$$

The fusion probability, then, is given by

$$P_{\text{fusion}} \propto \int_0^\infty f(E) P_0(E) dE \propto \int_0^\infty \exp(-E/kT - \bar{\eta}/E^{1/2}) dE$$

The integrand, which is a product of a falling exponential and a rising one as a function of increasing energy, is peaked function of energy, called the *Gamow peak*, located at an energy where the two exponentials have about the same value (fig. 3). The area under this peak gives the fusion probability. At temperatures typical of stellar interiors, the Gamow peak is located in the extreme tail of the thermal distribution, and a small change in temperature would cause a huge change in the area under the Gamow peak. The rate of stellar nuclear fusion processes thus turn out to be extremely sensitive to temperature. For example, Hydrogen burning rate has a temperature sensitivity of $\sim T^{5...10}$ over the relevant temperature range, while Helium burning rates could go as fast as T^{40}.

The extreme sensitivity to temperature has the effect that nuclear fusion takes place in stars in distinct, well-defined stages: the temperature at

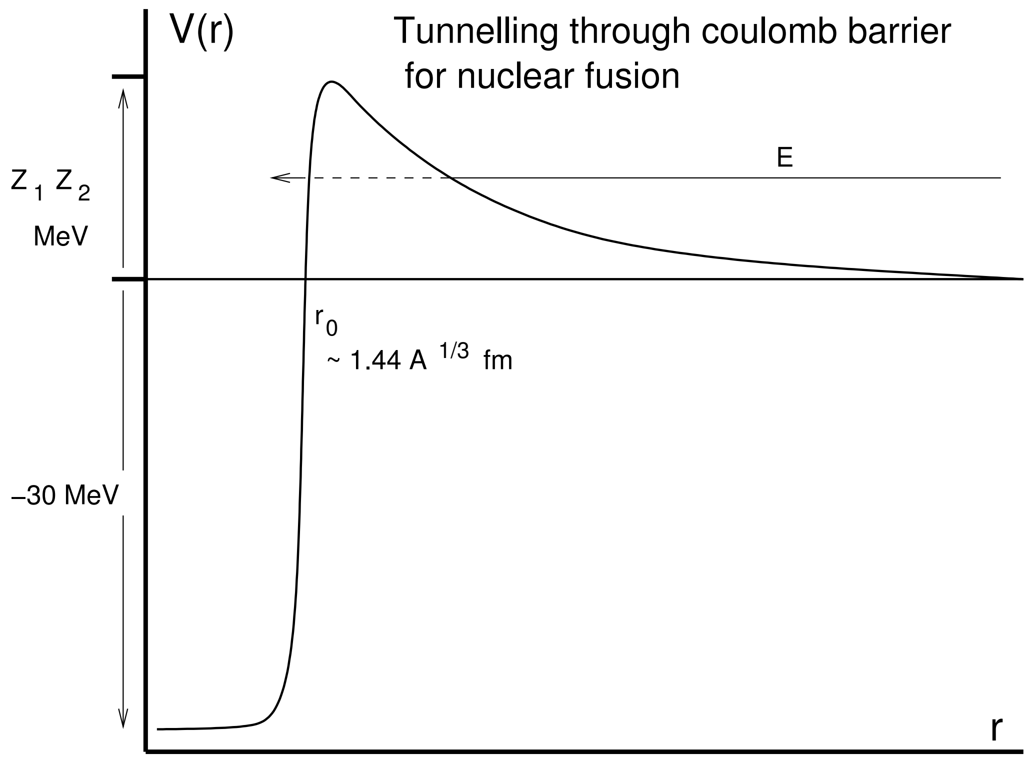

Figure 1: Sketch of the potential as a function of distance r between two fusing nuclei. Nuclear attraction dominates for $r < r_0$, and repulsive Coulomb barrier dominates at $r > r_0$. Here A is the mass number of the nucleus. A particle of energy E lower than the Coulomb barrier must tunnel through the barrier for fusion to be accomplished.

which one stage of burning takes place is not sufficient to ignite the subsequent stage. Once a burning stage is over, the stellar core must contract and raise the temperature further to ignite the next stage.

The first major nuclear burning stage in a star results in the fusion of 4 Hydrogen nuclei into a Helium nucleus. This is also the reaction which releases maximum amount of energy per unit mass, nearly 7 MeV out of 938 MeV for each nucleon (see fig. 2). This phase, therefore, lasts the longest and represents the "Main Sequence". The reactions involved in Hydrogen burning are as follows. At relatively low temperatures ($T < 1.5 \times 10^7$ K)

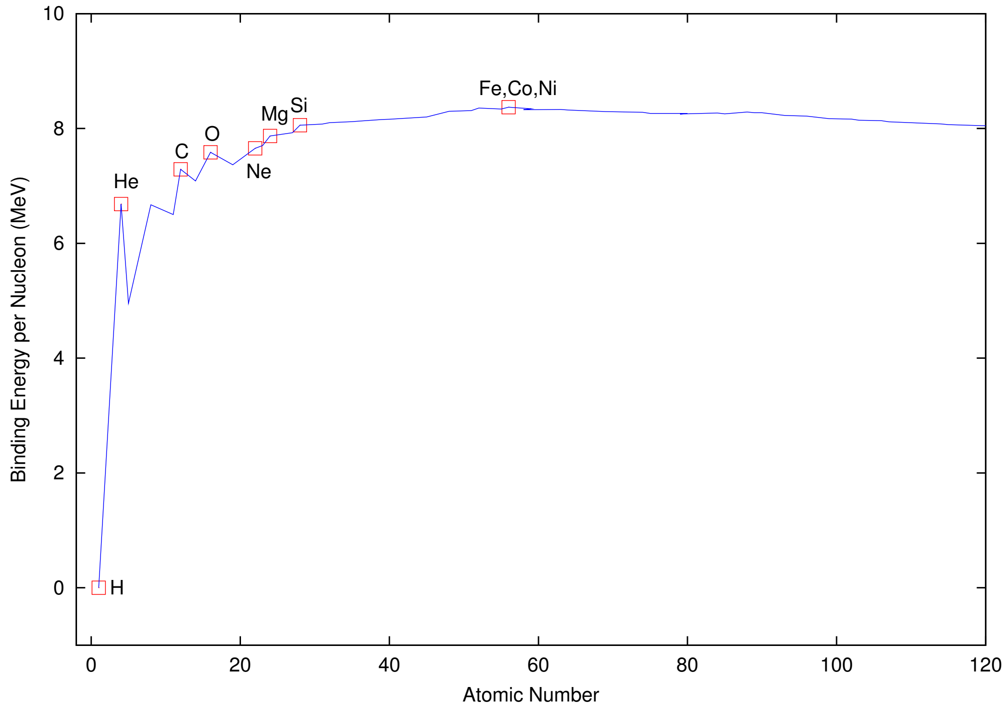

Figure 2: The nuclear binding energy curve: excess binding energy per nucleon over hydrogen nucleus. The major products synthesised/fuels consumed in different burning stages in the stellar interior are indicated by red squares

Hydrogen burns via the so-called "proton-proton chain" (*pp* chain):

$$\begin{aligned} {}^1\mathrm{H} + {}^1\mathrm{H} &\rightarrow {}^2\mathrm{H} + e^+ + \nu \\ {}^2\mathrm{H} + {}^1\mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \end{aligned}$$

Following which the reaction completes via one of three alternative routes. *pp*1:

$$ {}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2{}^1\mathrm{H} $$

*pp*2:

$$\begin{aligned} {}^3\mathrm{He} + {}^4\mathrm{He} &\rightarrow {}^7\mathrm{Be} + \gamma \\ {}^7\mathrm{Be} + e^- &\rightarrow {}^7\mathrm{Li} + \nu \\ {}^7\mathrm{Li} + {}^1\mathrm{H} &\rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} \end{aligned}$$

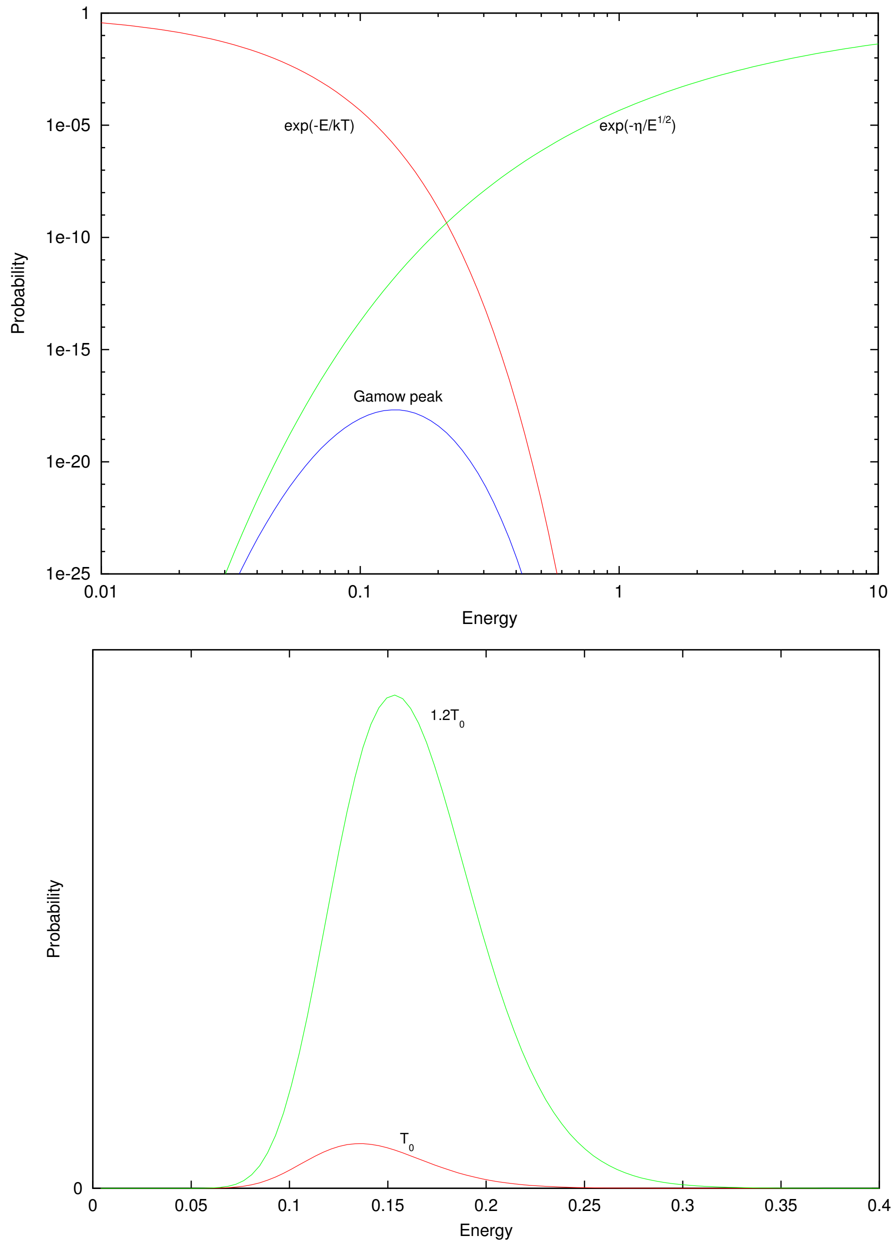

Figure 3: The Gamow peak and the two contributing exponentials: Boltzmann factor and the penetration factor are shown in logarithmic scale in the upper panel. The lower panel shows, in linear scale, the change in the Gamow peak if the temperature is increased by 20%. The reaction rate, proportional to the area under the Gamow peak, increases by more than a factor of 20.

*pp*3:

$$\begin{aligned}
{}^{3}\mathrm{He} + {}^{4}\mathrm{He} &\rightarrow {}^{7}\mathrm{Be} + \gamma \\
{}^{7}\mathrm{Be} + {}^{1}\mathrm{H} &\rightarrow {}^{8}\mathrm{B} + \gamma \\
{}^{8}\mathrm{B} &\rightarrow {}^{8}\mathrm{Be} + e^{+} + \nu \\
{}^{8}\mathrm{Be} &\rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}
\end{aligned}$$

At $T > 1.5 \times 10^{7}$ K the Carbon-Nitrogen-Oxygen cycle (*CNO* cycle) dominates, where pre-existing Carbon, Nitrogen and Oxygen play the role of catalysts:

$$\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} \\
&\quad \mathrm{or} \\
& {}^{16}\mathrm{O} + \gamma \\
{}^{16}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} &\rightarrow {}^{17}\mathrm{O} + e^{+} + \nu \\
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}
\end{aligned}$$

In either chain there are several important intermediate steps where neutrinos are emitted. The neutrinos from these different reactions carry different amounts of energy. It has therefore long been thought that by measuring the flux of neutrinos of different energies received from the Sun one can directly test the model of the Solar interior.

The earliest experiment of this category was started by Raymond Davis in the 1960s by putting 680 tons of cleaning fluid (C_2Cl_4) in a tank one mile underground in the Homestake Gold Mine. The capture of a solar neutrino by Chlorine would produce an Argon nucleus:

$$ {}^{37}\mathrm{Cl} + \nu \rightarrow {}^{37}\mathrm{Ar} + e^{-} $$

The Argon can be flushed out of the tank, and then counted. The Argon-37 nucleus captures an inner shell electron and leaves a Chlorine nucleus in an excited state, which de-excites by emitting photons that can be detected by conventional techniques. This technique is able to detect only the energetic neutrinos, emitted from the ^{8}B decay in *pp*3 chain.

The result of this experiment detected only one-third of the expected number of neutrino captures. This was a major puzzle, called the great "Solar Neutrino Problem", which remained unsolved for over three decades.

Following the Davis experiment other experiments were performed: one, called GALLEX, used 30 tons of Gallium as a target in a 100-ton aqueous solution of Gallium trichloride. This experiment had the sensitivity to detect the more abundant, but less energetic neutrinos from the *pp*1 chain. This was followed by the very large Super-Kamiokande experiment using 50,000 tons of purified water 1000 m underground in Japan. Both these experiments confirmed the deficit of Solar neutrinos.

What could be the reason for the discrepancy between the predicted and observed values? Either the Solar model, as constructed from stellar physics, is wrong, or there is some new physics about neutrino interaction that one does not understand.

In order to have an independent handle on the model of the Solar interior, concerted efforts have been made over the past decade to study Solar Oscillations - a technique that has acquired the name Helioseismology. Just as the study of seismic activity on the earth can reveal the state of its interior, so can it be for the Sun. This has been a large, coordinated, international effort through a formal group called GONG (Global Oscillation Network Group) of which India has been an active member. This effort has succeeded in measuring the Solar oscillations to unprecedented accuracy and, using them, to infer many details about the Solar interior. An important result of this exercise was the confirmation that the standard Solar model is very nearly correct.

The blame for missing neutrinos can thus only be put on some unknown piece of Physics. An important suggestion in this regard had been that since neutrinos come in three different flavours, the electron neutrino, the muon neutrino and the tau neutrino, there could be an oscillation between these flavours as neutrinos propagate through matter. Experiments designed to detect the electron neutrino will then detect only a fraction of the original number of neutrinos that started out. Such oscillations between flavours will only be possible, however, if the neutrino has a non-zero rest mass.

In the past few years there has been an increasing body of evidence that is may indeed be the case. Experiments on cosmic-ray-generated neutrinos as well as man-made neutrino beams at Kamiokande showed some evidence of neutrino oscillations. In June 2001, the Sudbury Neutrino Observatory, which is sensitive of all three flavours, announced the strongest evidence yet of the flavour oscillation of neutrinos. The Sudbury data leads one to infer a squared difference of rest mass between two neutrino eigenstates: $\Delta m^2 \sim 10^{-4}$ eV^2.

This is an example of how the pursuit of an Astrophysical problem can sometimes lead to fundamental discoveries in Physics. Indeed the contribution of Solar Neutrino experiments has been given the due recognition with the Physics Nobel Prize of 2002 awarded to both Raymond Davis of the Homestake experiment and Masatoshi Koshiba of Kamiokande.

A similar story also surrounds the next stage of nuclear burning in stars, namely the Helium Burning, which ignites at $T \sim 10^8$ K. The reaction reasponsible for this is called the *Triple Alpha reaction*:

$$
\begin{aligned}
{}^4\mathrm{He} + {}^4\mathrm{He} &\rightarrow {}^8\mathrm{Be} \\
{}^8\mathrm{Be} + {}^4\mathrm{He} &\rightarrow {}^{12}\mathrm{C} + \gamma
\end{aligned}
$$

The ^{8}Be nucleus formed in the first step is very unstable, it decayes back to two alpha particles in $\sim 10^{-16}$ s. Triple alpha reaction can succeed only if a third alpha particle is captured by the ^{8}Be nucleus within this short time. Fred Hoyle showed that at the prevailing temperatures this will have a negligible probability unless the capture is *resonant*, where the resulting ^{12}C nucleus has an excited level of energy very close to the sum of the energies of the ^{8}Be and ^{4}He nuclei. In presence of such a resonance, carbon can form in this excited state with higher than normal probability and can then deexcite down to the ground state by emitting photons. So in 1950-s Hoyle predicted that ^{12}C must have a resonant level at 7.6549 MeV. Later, experiments in the laboratory of Willy Fowler confirmed the existence of this resonant level. Fowler received the nobel prize in 1983 (sharing it with S. Chandrasekhar) for his contribution in the study of stellar nucleosynthesis.

The Helium burning reaction releases about one-tenth the energy of the Hydrogen burning phase, and the duration of this phase in the life cycle of the star is correspondingly shorter. While Carbon is the product of the triple-alpha reaction, once some Carbon has been produced then some Carbon nuclei can alpha-capture to produce Oxygen. The final product of the Helium burning is then almost equal amount of Carbon and Oxygen. Subsequent to helium burning, the later evolutionary stages, if reached, go through the following progression: Carbon burning to produce Oxygen, Neon and Magnesium. Neon burns to produce Oxygen, Magnesium and Silicon and then Oxygen burns to leave mainly Silicon as the product. Finally Silicon burns to produce ^{56}Fe, the most strongly bound nucleus. Each stage of nuclear burning produces less energy than the one previous (see fig. 2) to it. The progression through the burning phases thus get quicker with each stage. For example, in a $25M_\odot$ star, H-burning would last $\sim 5\times10^6$ y, He-burning $\sim 5\times10^5$ y, C-burning $\sim$ 60 y, Ne-burning $\sim$ 1 y, O-burning $\sim$ 6 months and Si-burning only $\sim$ 4 days.

Fusion reactions in stellar interior cannot proceed beyond ^{56}Fe, since after that it would be endothermic. Nuclei beyond the Iron-peak are synthesised mainly in neutron capture processes. Free neutrons are produced in small quantities in some of the nuclear reactions in the stellar interior, for example

$$\begin{aligned}
{}^{4}\mathrm{He} + {}^{13}\,\mathrm{C} &\rightarrow {}^{16}\mathrm{O} + {}^{1}\,\mathrm{n} \\
{}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{24}\mathrm{Mg} + \gamma \\
&\rightarrow {}^{23}\mathrm{Mg} + {}^{1}\,\mathrm{n} \\
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{31}\mathrm{Si} + {}^{1}\,\mathrm{n}
\end{aligned}$$

These free neutrons may be captured by other heavy nuclei to produce elements which cannot be synthesised by fusion. In ordinary circumstances in stellar interiors, the rate of production of free neutrons is relatively slow; and the element formed by neutron capture will have a chance to beta-decay if it is unstable, before capturing another neutron, if at all. The sequence of elements formed this way are called *s-process* elements (*s* for slow). In violent events such as supernova explosions, large flux of neutrons becomes available very rapidly, and under these conditions a nucleus may capture several neutrons before having a chance to beta decay. This produces a different sequence of elements, called *r-process* elements (*r* for rapid). Many of the relatively heavy nuclei are formed in the *r-process*.

We have discussed above the progression of nuclear reaction in the course of evolution of a star. Up to what stage of these reactions a star actually undergoes depends on the mass of the star.

Once the fuel in the hydrogen-burning core is exhausted, the thermal energy leaking out of the core is not replaced by energy generation, and this results in a contraction of the core. Gravitational energy released in the contraction (half of which is retained as internal thermal energy) will heat up the core and once the temperature reaches $\sim 10^8$ K, Helium burning will set in. However, in the process of contraction the core gets denser, causing the electron degeneracy pressure to rise. The thermal pressure rises as $\rho T \propto R^{-4}$, while the degeneracey pressure rises faster, as $\rho^{5/3} \propto R^{-5}$. We recall that on the main sequence the central density of a star is roughly inversely proportional to the square of its mass. So unless the star is sufficiently massive the degeneracy pressure will halt the gravitational contraction before the core temperature rises enough to burn Helium. The evolution of such a star will end here, leaving a Helium White Dwarf as the final product. A similar argument is applicable to every subsequent stage of nuclear burning, and detailed computations show that the zero-age main sequence mass of a star must exceed $\sim 10M_\odot$ for the nuclear reactions in the core to proceed all the way to iron-peak elements.

In the interval between one core burning stage and the next, as the core contracts and heats up, the unprocessed material above the core is also compressed and heated. This causes nuclear burning to be ignited in a shell around the inert core. When the inert Helium core contracts, Hydrogen burning begins in a shell overlying the core. The helium produced in the burning shell is added to the core, increasing its mass. As the core contraction proceeds, the shell source consumes more and more matter and becomes very luminous. The outer part of the star responds to this by expanding to a very large radius, decreasing its surface temperature and increasing its luminosity. A star in this state is called a "Red Giant". In the Hertzsprung-Russell diagram this corresponds to a track that moves to the right and then upwards. The increase in luminosity is not very significant for massive stars which are already very luminous in the main sequence. For them, the track moves nearly horizontally to the right (fig. 4). After subsequent core ignition, the core expands again when thermal pressure becomes important. This weakens the shell burning sources and the outer radius of the star contracts again. This leads to a reversal of the track direction in the HR diagram. A star may go through several such cycles corresponding to each nuclear burning stage, which will show up as 'loops' in the HR diagram.

If the core burning proceeds all the way to iron-peak elements then a contracting iron core will be surrounded by several shell burning sources: the innermost being that of Silicon burning, and progressively outwards Oxygen burning, Neon burning, Carbon burning, Helium burning and Hydrogen burning. This is analogous to an "onion-skin" structure (fig. 5).

Nuclear burning stages in the core that ignite while the core is highly degenerate can cause thermonuclear runaways: the rise in temperature due to nuclear burning does not increase the total pressure appreciably until kT nearly equals the Fermi energy E_F. This would require a large increase in temperature and the reaction rate (which is a strong function of temperature) would increase by many orders of magnitude.

The final remnants in different cases are

ZAMS mass	Final product type	Final product mass
$\lesssim 0.5M_\odot$	He White Dwarf	$\leq 0.45M_\odot$
$\sim 0.5 - 8M_\odot$	C-O White Dwarf	0.45–1.2$M_\odot$
$\sim 8 - 10M_\odot$	O-Ne-Mg White Dwarf	1.2–1.4$M_\odot$
$\sim 10 - 40M_\odot$	Neutron Star	$\sim 1.4 - 2M_\odot$
$\gtrsim 40M_\odot$	Black Hole	$\gtrsim 2.5M_\odot$

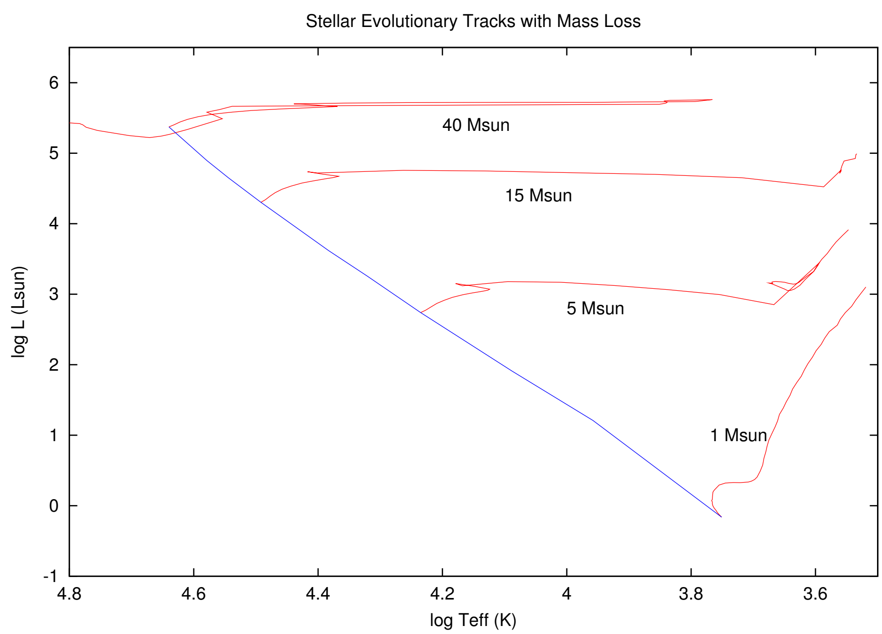

Figure 4: Example of evolution of stars in the Hertzsprung-Russell diagram. The evolutionary tracks displayed here have been computed by G. Schaller, D. Scharer, G. Meynet and A. Maeder of Geneva Observatory. Mass loss by stellar wind have been explicitly included in the computations.

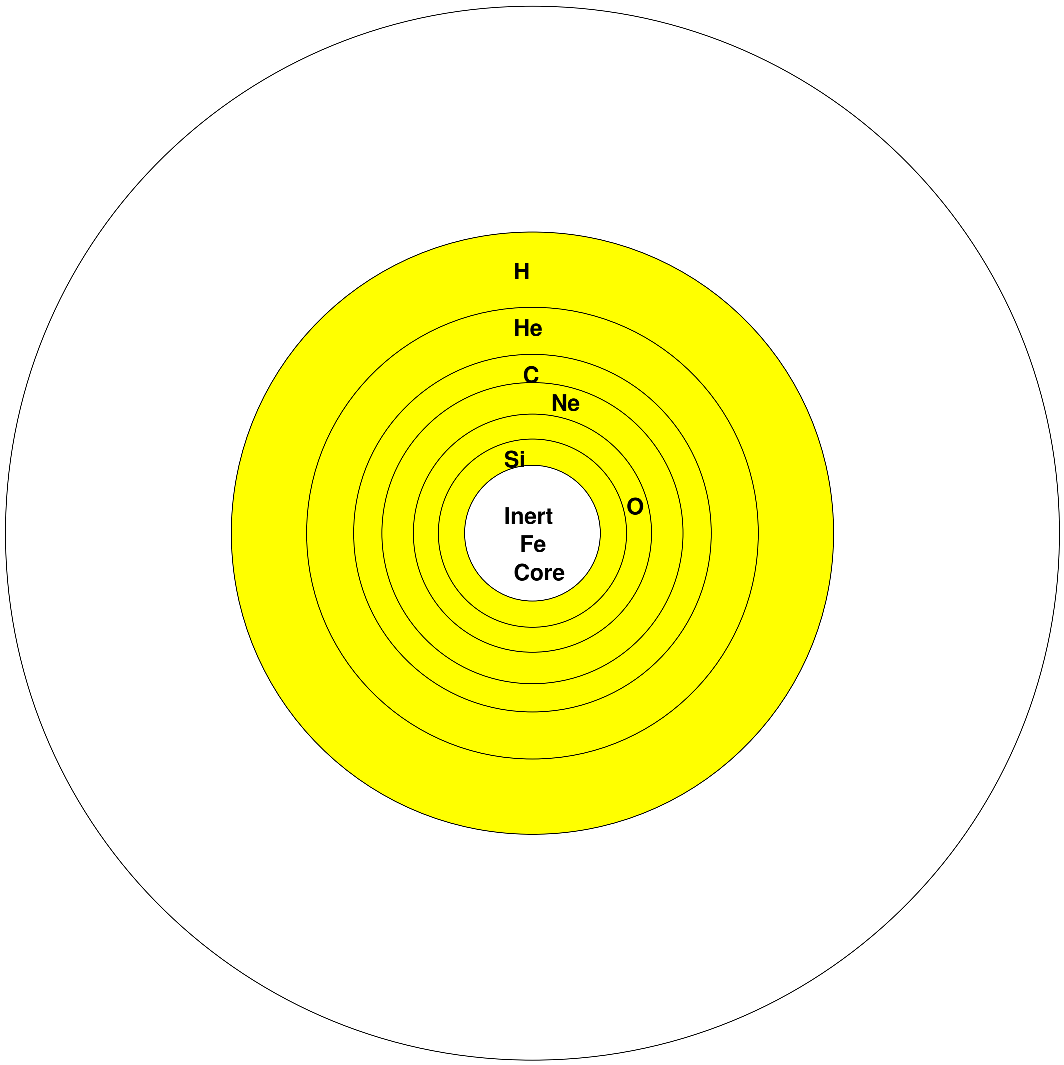

Figure 5: The "onion-skin" structure of a highly evolved star (schematic). The nuclear burning zones are shown in yellow, with the fuel indicated in each burning shell.

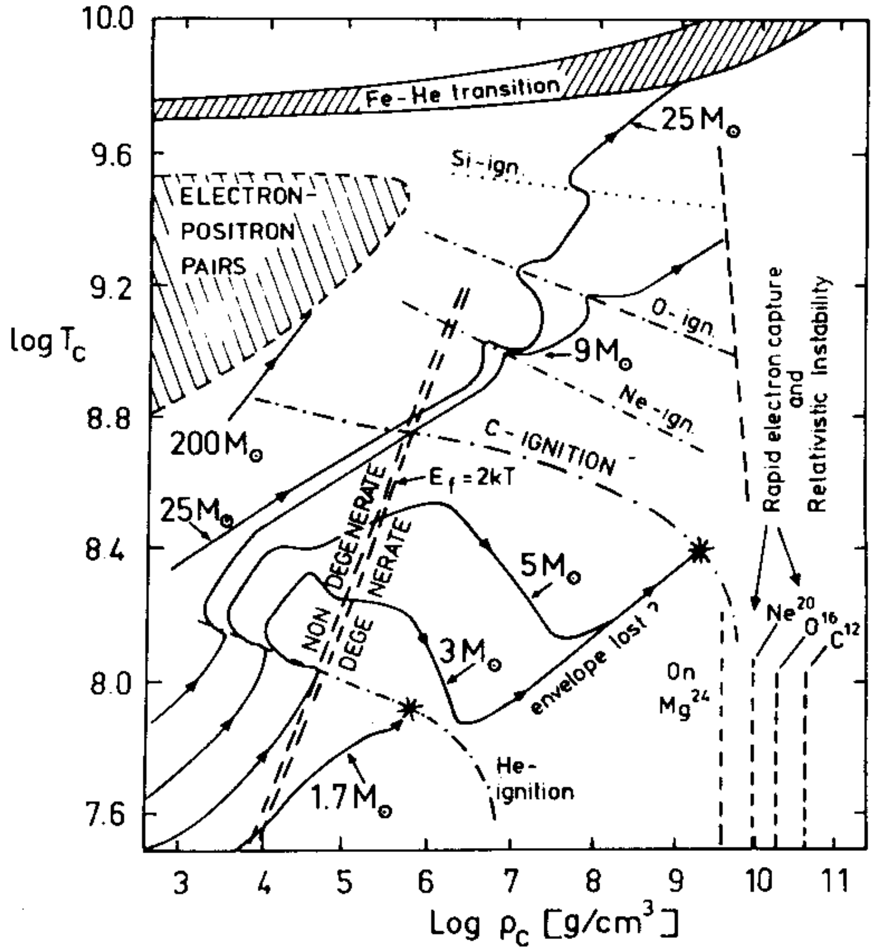

Figure 6: Evolution of central temperature and central density of stars. This diagram is taken from D. Bhattacharya and E.P.J. van den Heuvel (1992) Physics Reports, **203**, 1, and is based on computations by several authors, as referred to in the above article. The dot-dashed lines indicate the onset of different nuclear burning stages. Degenerate burning may lead to a thermonuclear runaway (indicated by asterisks). Degenerate helium burning gives rise to a helium flash which ejects most of the envelope of the star. Tracks shown for $3M_\odot$and $5M_\odot$stars are unlikely to proceed all the way to carbon flash if realistic mass loss is included–these stars are expected to end their lives as Carbon-Oxygen white dwarfs.