GRAVITATIONAL LENSING

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1 THE SCHWARZSCHILD LENS

Einstein's General Theory of Relativity predicts that a light ray which passes by a spherically symmetric mass distribution of total mass M at a distance ξ, is deflected by the Einstein angle

$$\alpha = \frac{4GM}{c^2\xi} = \frac{2R_s}{\xi} \tag{1}$$

provided the impact parameter ξ is much larger than the corresponding Schwarzschild radius.

$$R_s = \frac{2GM}{c^2} \tag{2}$$

To derive this result the linearized Schwarzschild metric suffices.The validity of (1) in the gravitational field of the Sun has been confirmed with radio-interferometric methods with an uncertainty of less than 1 percent.

To demonstrate the effect of a deflecting mass we show in the figure the simplest GL(gravitational lensing)configuration.A "point mass" M is located at a distance D_L from the observer O.The term "point mass" needs a little explanation.According to General Relativity, a static spherically symmetric body with Schwarzschild radius R_s has a geometrical radius R larger than $\frac{9}{8}R_s$,while for a static black hole ,$R = R_s$.Nevertheless,in lens theory, the term "point mass" is used whenever one is concerned with light rays deflected with impact parameters $\xi \gg R_s$ by a spherical object,irrespective of the (unobservable)behaviour of light rays with $\xi \sim R_s$. Now, the source is at a distance D_{LS} from the observer, and it is true angular separation from the lens M is β, the separation which would be observed in the absence of lensing.Due to the symmetry of the Schwarzschild lens, any ray travelling from the source to the observer is confined to the plane spanned by source,lens,observer.A light ray which passes the Lens at a distance ξ is deflected by α as given in the figure.The condition that this ray reach the observer is obtained solely from the geometry of the figure namely

$$\beta D_S = \frac{D_S}{D_L}\xi - \frac{2R_S}{\xi}D_{LS} \tag{3}$$

Here, D_{LS} is the distance of the source from the lens.In the simple case with a Euclidean background metric considered here $D_{LS} = D_S - D_L$;however, since GL occurs in the universe on large scales [among the hitherto detected GL events,most of the sources are QSOs(Quasi -stellar-objects)with redshift typically larger than 1],one must use a cosmological model.There ,"distance" does not have an unambiguous meaning,but several distances can be defined in analogy with Euclidean laws.The distances in Eqn (3) must then be interpreted as angular-diameter distances, for which in general$D_{LS} \neq D_S - D_L$.Denoting the angular separation between the deflecting mass and the deflected ray by

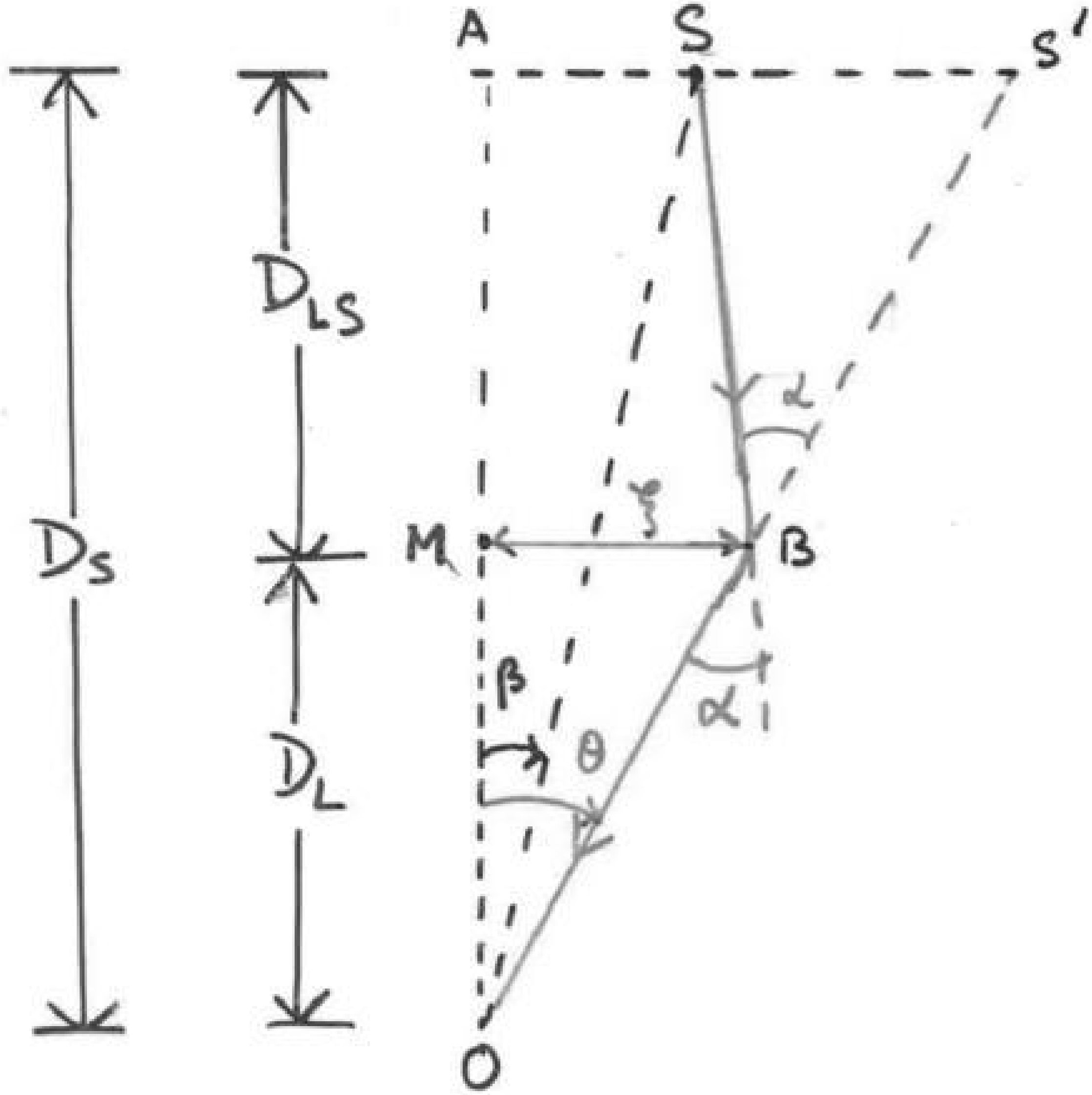
$$\theta = \frac{\xi}{D_L} \tag{4}$$

We obtain from (3)the **lens equation** as given by

$$\beta = \theta - 2R_S\frac{D_{LS}}{D_L D_S \theta} \tag{5}$$

We allow β and θ to have either sign.This exprn. clearly shows that the problem under consideration involves a characteristic angle $\alpha_0 = \sqrt{\frac{2R_S D_{LS}}{D_L D_S}}$ Also we define a characteristic length scale in the lens plane $\xi_0 = \alpha_0 D_L$ and a characteristic length scale in the source plane $\eta_0 = \alpha_0 D_S$ Writing (5) as

$$\theta^2 = \beta\theta + \alpha^2 \tag{6}$$


A
S
S'
D_{LS}
D_S
M
ξ
B
α
β
θ
D_L
α
O

We obtain

$$\theta_{1,2} = \frac{1}{2}(\beta \pm \sqrt{4\alpha_0^2 + \beta^2} \tag{7}$$

The angular separation between the images is $\Delta\theta = \sqrt{4\alpha_0^2 + \beta^2} \geq 2\alpha_0$.(According to the terminology of optics the deflector **does not act as a lens which produces images-in general the rays are not focussed at the observer,i.e. a GL does not have a focal length**.However it has become customary to abuse the terms lens and image in the context of GL theory). Thus the lens equation always has two solutions of opposite sign.This means that the source has an image on each side of the lens.It can be shown that *the two images are of comparable brightness only if β and hence $|\theta|$ is of order α_0.* **Einstein Rings:**A special situation arises if source,lens and observer are colinear,i.e.$\beta = 0$.In this case there is no preferred plane for the light rayrs to propagate ,but the whole configuration is rotationally symmetric about the line-of-sight to the lens.For $\beta = 0$ the solutions (6) become $\theta_{1,2} = \pm\alpha_0$.Hence due to symmetry the whole ring of angular radius $\theta = \alpha_0$ is a solution to the lens equation.Such images are called Einstein rings. **Drawbacks of Schwarzschild model**:Even if the deflector is a star, its field is distorted either because the star is a part of a galaxy which provides a tidal field,or a disturbance is due to galaxies lying near the line-of-sight to the source which produces additional distortions. *But this simple model is useful because it provides relations between the lens mass,observer to lens distance,observer to source distance,$\theta_{1,2}$ etc. which are of the same order of magnitude as for realistic lenses.*

2 THE GENERAL LENS

We refer to the same figure as was given for the Schwarzschild lens,but here the mass distribution is not spherically symmetric.the plane perpendicular to the plane of the paper containing the source S is called the source plane.Similarly the lens plane is also defined.The straight line through O and L is called the optical axis.The separation of the light-ray from the optical axis described by the two-dimensional vectorξin the lens plane.$\theta, \alpha, \beta$are also described by 2-D vectors in the lens plane.Then (5) goes over to

$$\beta = \theta - \frac{D_{LS} D_S}{\alpha(\xi)} \tag{8}$$

If $|\eta| = D_S \beta$, denotes the position of source w.r.t optical axis,then we have

$$\eta = \frac{D_S}{D_L}\xi - D_{LS}\alpha(\xi) \tag{9}$$

From (),if image position θ and the deflection law for the lens -$\alpha(\xi)$ is given, true position β is determined. But,**the typical problem in GL theory is to invert the lens equation,i.e. to find all images of a source for a given matter distribution , or to find, for a given image position, a suitable matter distribution .**

3 observing GL systems

To classify a set of observed objects as GL system, we must have 1.at least 2 images close together on the sky. 2.flux ratios in different spectral bands that are same for all images. 3.red shifts that are same for all images 4. a possible lens in the vicinity of the images but at redshift smaller than that of the images. 5,temporal variations in different images that are correlated(in case of sources of variable brightness)

Stars crossing the beam that form images due to a lens such as a galaxy may mask intrinsic fluctuations.This effect is called Microlensing.It also leads to variations of the point source magnifications.