Appendix B

Polarized Light in Astronomy

B.1 Introduction

Light can get polarized in two ways - either due to the mechanism of emission itself or due to the interaction with the medium through which it propagates. Polarization at the source may be either because the charges which emit the radiation are constrained in their degrees of freedom, for example by an electric or magnetic field, or because the radiation is due to stimulated emission. On the other hand, the medium of propagation affects light in three ways, namely absorb, retard and scatter – all three can affect the state of polarization of light passing through it. In a given situation, any or all of these processes might be at work and what is measured is the result of their complex interplay. But often these different processes leave different imprints on the radiation, in terms of its properties like intensity, spectral content and polarization state. In many situations, these processes are such that they result in time variability of the properties of light. Frequently, the same underlying physical process manifests quite differently visa vis the radiation properties, under different viewing perspectives. Polarimetric techniques attempt to glean information about astrophysical phenomena by studying the dependence of polarization properties of light on the physical attributes of the source and the intervening medium.

In the following sections we will illustrate some polarization mechanisms with examples relevant to astronomy. The emphasis of this discussion will not be on exact details of the actual astronomical situation, but on demonstrating, with one mathematical formalism, how the basic physical mechanism in each case naturally results in the production of polarized light. The concepts of polarized and unpolarized radiation and their mathematical representation (the Stokes vector formalism) has been discussed briefly in appendix A.

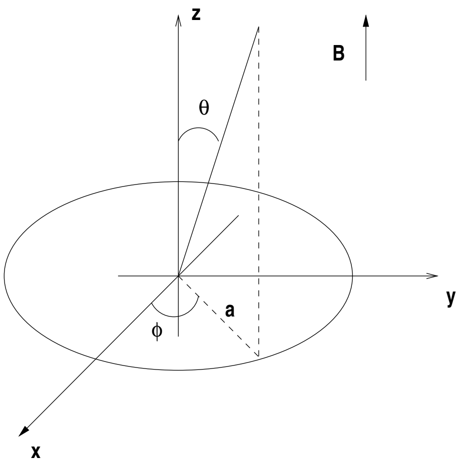

Figure B.1: The angles θ and ϕ with reference to the Cartesian coordinate system.

B.2 Polarization at the source

B.2.1 Synchrotron Radiation

Synchrotron radiation is a classical example for polarized emission of radiation resulting because of a magnetic field constraining the motion of the radiating charges. To see this, consider a charge q of mass m moving with a velocity v perpendicular to a uniform magnetic field of strength B along the $z - axis$ (see Fig. B.1). It describes circular motion of radius a and angular frequency ω_0 given by

$$a = \frac{mv}{qB} \qquad \text{and} \qquad \omega_0 = \frac{qB}{m} \,. \tag{B.1}$$

This motion can be described in a Cartesian coordinate system using the complex notation as

$$x = ae^{i\omega_0 t} \qquad \text{and} \qquad y = ae^{i(\omega_0 t - \frac{\pi}{2})}, \tag{B.2}$$

so that the acceleration along the x and y axes are

$$\ddot{x} = -a\omega_0^2 e^{i\omega_0 t} \qquad \text{and} \qquad \ddot{y} = a\omega_0^2 e^{i(\omega_0 t + \frac{\pi}{2})} \,. \tag{B.3}$$

Equation A.6 of Appendix A gives expressions for the electric field radiated by an accelerated charge. Substituting Eq. B.3 in Eq. A.6, we get the field at a far off point as

$$E_\theta = \frac{-q}{c^2 R} a\omega_0^2 \cos\theta e^{i\omega_0 t} \qquad \text{and} \qquad E_\phi = \frac{q}{c^2 R} i a\omega_0^2 e^{i\omega_0 t} \tag{B.4}$$

where c is the velocity of light in vacuum and R is the distance from the charge to the observer. Substituting Eq. B.4 in the definition of Stoke's vector given in Eq. A.7, we obtain,

$$\mathbf{S} = \left(\frac{q}{c^2 R} a\omega_0^2\right)^2 \begin{bmatrix} 1+\cos^2\theta \\ \sin^2\theta \\ 0 \\ 2\cos\theta \end{bmatrix} \,. \tag{B.5}$$

This represents elliptically polarized light in general, but reduces to simpler forms for certain special position angles of the observer. For example, if the observer is looking along the z-axis so that $\theta = 0$, the Stoke's vector reduces to that of right-hand circularly polarized light. On the other hand for an observing angle corresponding to $\theta = 90°$, the light will appear linearly polarized with the electric field vector contained in the $x - y$ plane.

Though the above picture shows how polarization can result from the emission mechanism itself, it is too simplistic to represent real astronomical situations. For example, it ignores the fact that synchrotron radiation occurs under conditions where relativistic effects cannot be neglected. The main difference which relativity makes is that the radiation will be highly beamed in the direction of motion of the charge. Another simplifying assumption which has been made above is that the charge is moving perpendicular to a uniform field. In reality, however there will be a distribution of velocities and variation of magnetic field due to which the observed radiation will be only partially polarized.

There are quite a few instances in astronomy where synchrotron radiation is important. The interstellar medium of our galaxy and other galaxies are transparent to radiation at centimeter wavelengths and hence synchrotron radiation at these wavelengths can be used to study the structure of magnetic fields as projected in the plane of the sky. For excellent recent reviews refer to Wielenbinski (1992) and Kronberg (1994). The distant quasars and active galaxies constitute another class of objects that have been studied using synchrotron radiation. Magnetic fields of the order of 10^8 G produces synchrotron radiation at optical wavelengths under suitable conditions. Such strong fields are not very common, but they do exist in objects like pulsars. One of the objects which have been studied with synchrotron radiation at almost all wavelengths is the Crab nebula in Taurus constellation (Felten 1974).

B.2.2 Masers and Zeeman effect

Astronomical masers constitute another source of polarized radiation. But the observed polarization in these sources results from the presence of magnetic fields producing Zeeman splitting of the magnetic quantum levels and is not a consequence of stimulated emission (Goldreich et. al. 1973, Elitzur 1996 and references therein). In a classical framework the Zeeman effect can be understood as a result of the precession of the orbits of electrons oscillating within atoms in the presence of a magnetic field (see Fig. B.2). The equations of motion of such an electron with a natural frequency ω_0, can be written as (Collet 1993)

$$\begin{aligned} x(t) &= \frac{A}{2}\sin\chi\,(\cos\omega_+ t + \cos\omega_- t) \\ y(t) &= \frac{A}{2}\sin\chi\,(\sin\omega_+ t + \sin\omega_- t) \\ z(t) &= A\cos\chi\cos\omega_0 t \end{aligned} \tag{B.6}$$

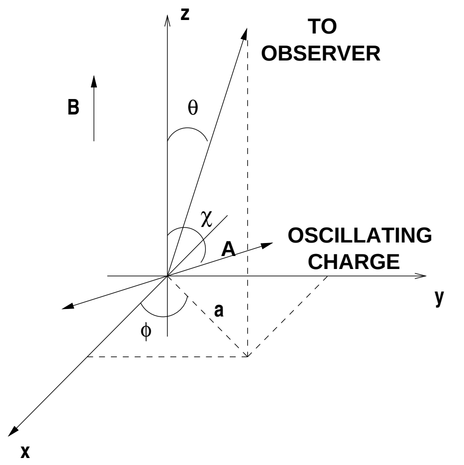

Figure B.2: Motion of a bound charge in a constant magnetic field.

where $\omega_\pm = \omega_0 \pm \omega_{\rm L}; \omega_{\rm L} = \frac{eB}{2mc}$, being the Larmor precession frequency. Differentiating Eq. B.6 twice, substituting in Eq. A.6 and deriving the Stoke's vector according to Eq. A.7, we get

$$\mathbf{S} = \frac{2}{3}\left(\frac{qA}{2c^2R}\right)^2 \left(\omega_-^4 \begin{bmatrix} 1+\cos^2\theta \\ -\sin^2\theta \\ 0 \\ -2\cos\theta \end{bmatrix} + \omega_0^4 \begin{bmatrix} 2\sin^2\theta \\ 2\sin^2\theta \\ 0 \\ 0 \end{bmatrix} + \omega_+^4 \begin{bmatrix} 1+\cos^2\theta \\ -\sin^2\theta \\ 0 \\ 2\cos\theta \end{bmatrix} \right) . \quad \text{(B.7)}$$

Eq. B.7 clearly shows the nature of polarization of the three levels predicted by the classical model of Lorentz. Considering the particular case of observing along the magnetic field direction ($\theta = 0$), the Stoke's vector reduces to two lines ω_- and ω_+, which are left and right circularly polarized respectively (*longitudinal* Zeeman effect). On the other hand, an observer at $\theta = 90^\circ$ observes all the three lines, with ω_- and ω_+ vertically polarized and ω_0 horizontally polarized (*transverse* Zeeman effect). Finally, in the absence of a magnetic field (ie. $\omega_- = \omega_0 = \omega_+$), we get unpolarized light.

Zeeman effect is one of the most widely used technique to trace the magnetic field distribution in a wide variety of astronomical objects particularly at radio wavelengths (the separation between the spectral lines is of the order of $\frac{qB}{m}$) – solar magnetic fields (Stenflo 1994), 'spots' on magnetic A_p stars, magnetic white dwarfs (Landstreet 1992), Galactic neutral hydrogen clouds, Galactic molecular clouds (Myers and Goodman 1988) etc. are some examples.

B.3 Effects of the propagating medium

B.3.1 Absorption and Retardance

Polarization due to absorption and retardance is a manifestation of anisotropy in the imaginary and real parts respectively of the complex refractive index of the medium of propagation.

Some media are anisotropic in nature so that they absorb one polarization component of light in preference to the other. This effect known as dichroic absorption is mostly attributed to the presence of slightly aspherical dust grains in the interstellar medium which are on an average aligned with their short axes parallel to an ambient magnetic field (see Sec. B.3.2 below for more details). Thus they are capable of introducing polarization into incident unpolarized light and also of changing the state of polarization if the incident light is already polarized. Polarization due to true dichroic absorption becomes non-negligible only in the wavelength range between $5 - 60\ \mu m$ (Hildebrand 1988), while at shorter wavelengths it is completely dominated by scattering.

The retardance of the medium affects the polarization state of the light only if it is birefringent. In general, in a birefringent medium the refractive indices for the two eigen modes of polarization will be the same only for a particular direction of propagation. For all the other directions, the two orthogonal components will have different refractive indices and their values will be different in different directions. As light travels along any such direction, the equivalent path length will be different for the two polarization components. In other words, the two components will have different phase retardances and according to Eq. A.4, the polarization state of the light changes. However, it is to be noted that a birefringent medium can only alter the nature of polarization of incident light which is already polarized; it cannot polarize unpolarized light. For example, linearly polarized light propagating through a linearly birefringent medium will partly be converted to circular polarization by this effect.

Many materials, among which of particular interest for us are astrophysical plasmas, become circularly birefringent in the presence of a magnetic field. It is easy to understand the effect of such a medium on polarized light if we express the propagating electric field in terms of circularly polarized eigen vectors as follows :

$$E = \left(A_{\rm L} e^{i\phi_{\rm L}} + A_{\rm R} e^{-i\phi_{\rm R}} \right) e^{i(\omega t - kz)} \tag{B.8}$$

where $(A_{\rm L}, \phi_{\rm L})$ and $(A_{\rm R}, \phi_{\rm R})$ are positive constants representing the amplitude and phase of the left and right circularly polarized components respectively. For linearly polarized light $A_{\rm L} = A_{\rm R} = A$ and the orientation of the electric field vector is given by $\phi_{\rm L} - \phi_{\rm R}$. As light traverses through the medium, an additional phase difference is introduced between the two circular polarization components. Thus the position angle of the linearly polarized light rotates as it propagates through a circularly birefringent medium. The amount of rotation is directly proportional to the component of magnetic field along the direction of propagation and also the distance which the light travels through the medium. This effect is known as Faraday rotation. Such rotation of the polarization vectors of radiation coming from pulsars and other extra-galactic radio sources have been used extensively to map the magnetic field in our own Galaxy.

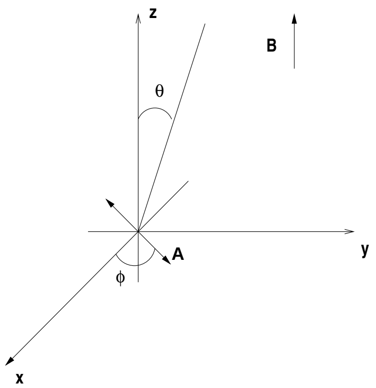

Figure B.3: Electron motion confined to the $x-y$ plane due to an electric field incident in the z direction.

B.3.2 Scattering

A major concern of this work is the study of polarization produced by scattering in astrophysical situations. Hence we will discuss this process in some detail so that the characteristics of the astrophysical phenomena which fall under this category are brought out.

Polarization due to Scattering Geometry

Scattering of light is almost always accompanied by its polarization. This is a direct consequence of the transverse nature of radiation due to which it cannot have components along the direction of propagation. To illustrate this let us consider an optically thin cloud of small, isotropically distributed spherical particles being illuminated from one side (say, along z-axis) by a beam of unpolarized radiation. Due to the transverse nature of radiation, the charges in the cloud will be set into oscillation within the x-y plane, as shown in Fig. B.3. The equation of motion of any one such charge can be written as

$$\begin{aligned} x(t) &= A\cos\phi e^{i\omega_0 t} \\ y(t) &= A\sin\phi e^{i\omega_0 t} \\ z(t) &= constant \end{aligned} \tag{B.9}$$

where ϕ is the angle between the x-axis and the direction of oscillation of the charge. Differentiating Eq. B.9 twice, substituting in Eq. A.6 and deriving the Stoke's vector according to Eq. A.7, we get

$$\mathbf{S} = \left(\frac{qA}{c^2R}\right)^2 \omega_0^4 \begin{bmatrix} \sin^2\phi + \cos^2\phi\cos^2\theta \\ \sin^2\phi - \cos^2\phi\cos^2\theta \\ 2\sin\phi\cos\phi\cos\theta \\ 0 \end{bmatrix} . \tag{B.10}$$

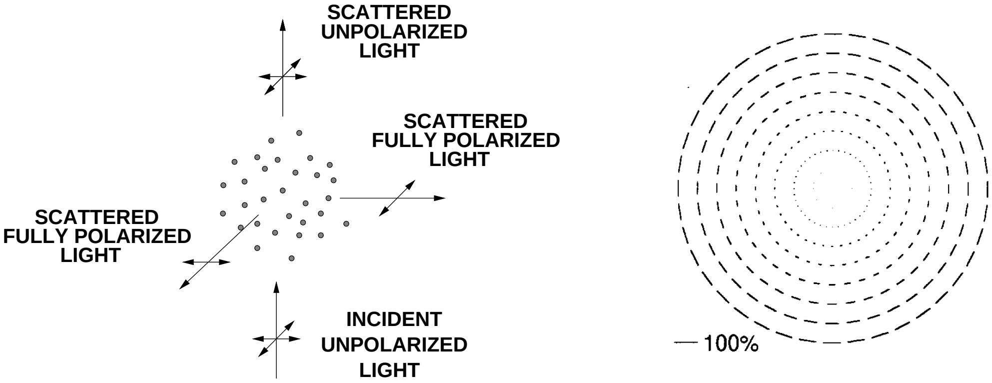

Figure B.4: Polarization of radiation due to scattering geometry. The figure on the right-hand side (adapted from Matsumura & Seki 1996) depicts how polarization varies as a function of θ and ϕ. Each concentric circle correspond to a θ value (0°at the centre to 72°for the outermost circle) and the azimuth angle correspond to ϕ value.

This shows that the scattered beam will be fully polarized with the polarization position angle depending on the azimuth angle ϕ at which the charge is set to oscillate and also on the viewing angle θ of the observer. But here, we are actually interested in determining how the medium of scattering centers as a whole affects the polarization properties of the light passing through it (see Fig. B.4). For this we make use of the property that the Stoke's vector of a beam made by combining many independent beams is just the sum of Stoke's vectors of the components. Thus the polarization state of the radiation scattered by the ensemble of scattering centers is obtained by integrating each of the Stoke's parameters in Eq. B.10 above, over all values of ϕ between 0 and 2π. Thus for the entire cloud

$$\mathbf{S}_{\text{cloud}} = \pi \left(\frac{qA}{c^2 R}\right)^2 \omega_0^4 \begin{bmatrix} 1+\cos^2\theta \\ 1-\cos^2\theta \\ 0 \\ 0 \end{bmatrix} . \tag{B.11}$$

We see that the radiation scattered in this way is always linearly polarized except for $\theta = 0°$ and $\theta = 180°$ in which cases it becomes unpolarized. The beam will be fully polarized for an observer at $\theta = 90°$. The polarization position angle is always within the $x - y$ plane and also orthogonal to the scattering plane (the plane defined by the directions of the incident radiation and the scattered radiation). At observing angles other than 90° and 180°, the scattered radiation is partially polarized. The degree of polarization is given by

$$\Pi = \frac{1-\cos^2\theta}{1+\cos^2\theta} \tag{B.12}$$

The geometry of polarized radiation produced through scattering has been employed in the study of a wide variety of astronomical phenomena. In the interstellar medium of our Galaxy, there are some clouds of gas and dust which do not emit much radiation of their own, but often scatters light falling on them from nearby bright

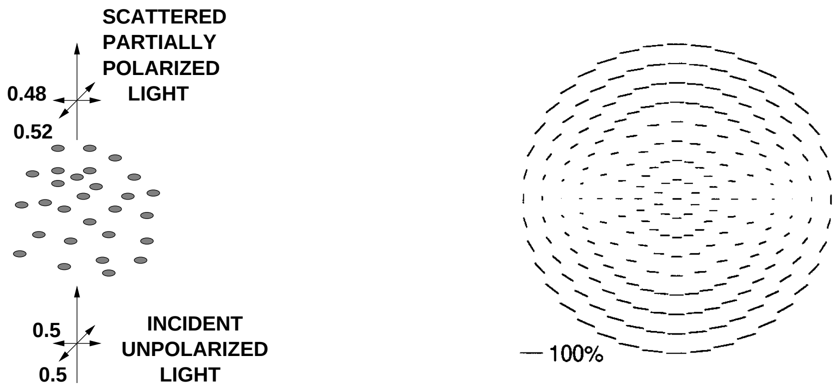

Figure B.5: Polarization of radiation due to dichroic scattering. The figure on the right hand side (adapted from Matsumura & Seki 1996) depicts how polarization varies as a function of θ and ϕ. Each concentric circle correspond to a θ value (0°at the centre to 72°for the outermost circle) and the azimuth angle correspond to ϕ value.

objects. These are, in general, called reflection nebulae. Frequently the illuminating sources of these objects are hidden from direct view by dust in the line of sight and can be studied only by their light reflected by the cloud. In such cases a powerful way of identifying the source which illuminates the cloud is by looking at the centrosymmetric pattern of polarization vectors of the reflected light (Scarrott 1991, Walther 1990). Polarimetry can also yield information regarding the distribution of gas and dust, the size and composition of the scatterers etc. (McLean 1985).

For several years astronomers have been trying to arrive at a common unifying basis for the apparently very dissimilar objects usually lumped under the name of active galaxies. A basic assumption behind these unifying schemes is that these objects are all essentially the same, but with very complex and anisotropic structural geometry and kinematics, that they appear very dissimilar under different viewing perspectives. The greatest difficulty with these models is the inability of the current observational techniques of resolving these objects at sufficiently detailed scales. Polarimetry of light scattered by small dust grains or electrons close to the nuclei of these objects have been employed extensively to discern their geometry - for example the hyperluminous IR galaxies (Hines et. al. 1995), Broad Absorption Line Quasi-Stellar Objects (Goodrich & Miller 1995) etc.

Polarization due to dichroic scattering

There is yet another important mechanism of producing polarized radiation through scattering (see Fig. B.5). More than four decades ago Hall (1949) and Hiltner (1949) observed that light from reddened stars at optical wavelengths is polarized. Clearly this cannot be explained by the scattering mechanism described above because in that case according to Eq. B.12, the forward scattered light remains essentially unpolarized.

A new thoery was put forward by Davis and Greenstein (1951), in which they proposed that the interstellar medium consists of slightly aspherical dust grains which contain paramagnetic material, and are initially spinning in random orientations, due to collisions with the gas, at angular velocities of the order of $10^5 - 10^6$ rad/s. They also proposed the presence of an ambient magnetic field in the interstellar medium, the idea of which was supported by Galactic synchrotron radiation observations carried out at about the same time. Due to the paramagnetic nature of the dust grains, the internal magnetic dipoles in them try to align with the external field, but they fail to do so completely because of the spinning of the grains. The internal field therefore lags behind the external one and develops a component orthogonal to it, as a result of which the grain experiences a torque which tends to dampen the rotation. Now, the total rotational energy about any spin axis is comparable to the thermal energy of the gas, ie $\frac{1}{2}I\omega^2 \simeq kT_{\mathrm{gas}}$. Also for a grain of dimension $2l$, the moment of inertia I is proportional to l^2. Therefore the spin angular frequency ω will be larger for rotation about the long axis of the grain. Thus eventually the damping *torque tends to make the short axis of the grain the axis of rotation, which is still very rapid, and to set this short axis along the magnetic field* (Davis and Greenstein 1951). In effect, the short axis, the angular momentum vector and the ambient magnetic field, all get aligned. As light of wavelength comparable to the grain sizes, propagate through such a medium, the electric field component along the long axes get preferentially scattered. An observer looking at a source shining through the medium will find it partially linearly polarized parallel to the direction of short axis, ie. the magnetic field. Though it was soon realized that the mechanism of Davis & Greenstein does not achieve grain alignment efficiently enough to explain the observed polarization, it is variants of this basic theme itself which are still used to explain the observations.

The cold, dense molecular clouds embedded in the tenuous interstellar medium are sites of very interesting dynamics, particularly associated with star formation and often have very intricate features. Magnetic fields are supposed to play an important role in the dynamics of these clouds. Though it is difficult to measure the strength of these magnetic fields, their orientation as projected in the plane of the sky can be determined using polarimetry of starlight scattered by the dichroic dust grains in these clouds (see Chap. 6).

On larger scales, the distribution of the magnetic field in the Galaxy has been mapped using polarimetry of starlight scattered by the interstellar medium (Mathewson & Ford 1970). The large scale distribution of magnetic fields and the distribution of dust in relation with the spiral arm structure of galaxies has been another subject of polarimetric studies. Presence of dust lanes and other features in interacting and active galaxies also manifest themselves through their polarimetric signatures.

Wavelength dependence of polarization by dichroic scattering

The optical depth due to a dust cloud of size L, made up of grains of size a is given by $\tau = n_{\mathrm{d}}\pi a^2 QL$, where n_{d} is the number of grains per unit volume and Q is a factor of order unity, called the scattering efficiency. If the dust grains are aspherical and their long axes are aligned with each other, two polarization components of light, parallel to the long and short axes of the grains, will undergo different amounts of scattering. Hence the intensities of the polarization components emerging from the cloud, will be

$$I_l = I_l(0)e^{-\tau_l} \qquad \text{and} \qquad I_s = I_s(0)e^{-\tau_s} \ , \tag{B.13}$$

where $I_l(0)$ and $I_s(0)$ are the intensities of the polarization components along the long and short grain axes respectively, of the incident light and τ_l and τ_s are the corresponding optical depths. For unpolarized incident light, $I_l(0) = I_s(0) = I(0)$ and therefore

$$I_l \pm I_s = I(0)[e^{-\tau_l} \pm e^{-\tau_s}] = I(0)[\tau_l \pm \tau_s] \ , \tag{B.14}$$

provided $\tau_l, \tau_s \ll 1$. But since, $\tau_l \propto Q_l$ and $\tau_s \propto Q_s$ (Q_l, Q_s being the scattering efficiencies of the grains for the two polarization components), we obtain the degree of linear polarization

$$p = \frac{I_l - I_s}{I_l + I_s} = \frac{Q_l - Q_s}{Q_l + Q_s} \ . \tag{B.15}$$

Thus, the polarization of light by dichroic scattering depends on the difference between the scattering efficiencies of the dust grains, for the two polarization components parallel to their long and short axes. In fact, computations based on Mie theory show that the wavelength at which Q_l - Q_s becomes maximum, is directly related to the grain dimensions. For long, dielectric cylinders of radius a and refractive index n, the wavelength of maximum polarization λ_{max} is given by (Whittet 1992a)

$$\lambda_{max} = 2\pi a(n-1) \ . \tag{B.16}$$

Multi-wavelength polarization studies of various lines of sight in the Galaxy show that the value of λ_{max} ranges between 0.4 μm to 0.8 μm with an average value of 0.55 μm. Earlier observations (eg. Serkowski, Mathewson & Ford 1975) at optical wavelengths led to the coining of an empirical law for the wavelength dependence of interstellar polarization which came to be known as the Serkowski law,

$$\frac{p_\lambda}{p_{max}} = \exp[-1.15\ln^2(\frac{\lambda_{max}}{\lambda})] \ , \tag{B.17}$$

where p_λ is the degree of polarization at wavelength λ and p_{max} is that at λ_{max} (see Fig. B.6).

Further observations at wavelengths up to 2.2 μm suggested that the constant 1.15 in Eq. B.17 be replaced by an expression of the form $c_1\lambda_{max} + c_2$ (Wilking et al 1982), where λ_{max} is in μm. The current best estimates of c_1 and c_2 are given by (Whittet et. al. 1992b)

$$c_1 = 1.66 \pm 0.09(\mu m)^{-1} \qquad \text{and} \qquad c_2 = 0.01 \pm 0.05. \tag{B.18}$$

The replacement of the constant with the above linear expression implies a systematic decrease in the width of the polarization curve with increasing λ_{max}. As further

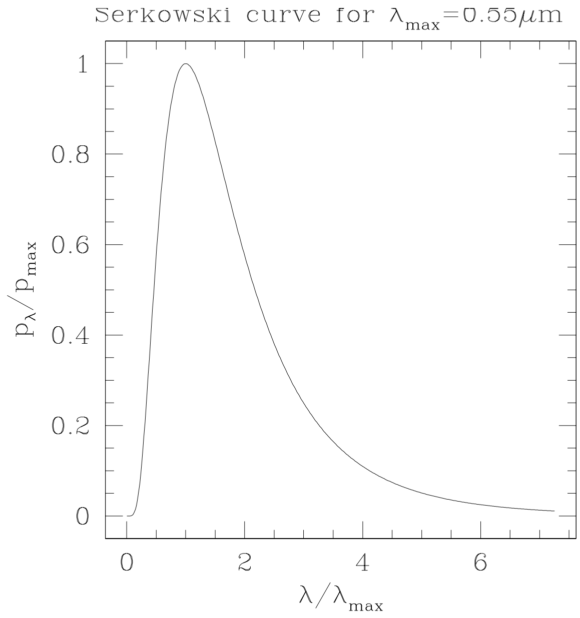

Figure B.6: Wavelength dependence of interstellar polarization

observations in the wavelength range 2-5 μm became available, it is seen that the observed polarization at these longer wavelengths shows significant departures from that predicted by the above law. In this regime, a power law of the form

$$p_\lambda = p_1 \lambda^{-\alpha} \tag{B.19}$$

fits the observations better, with $\alpha = 1.8 \pm 0.2$. This behavior is consistent with the observed power-law nature of the extinction curve at these wavelengths, given that polarization results from differential extinction.

Not much progress has been made in extending the polarization curve to the ultraviolet end of the wavelength regime because of the difficulty of ground based observations at these wavelengths. The situation is changing now, with the advent of space based telescopes; but only about fourteen lines of sight has been investigated in the UV so far. There are indications from these observations (Clayton 1995) that the interstellar UV polarization characteristics fall into two categories – one which follows an extrapolation of the Serkowski relation into the UV and the second which shows an excess polarization compared to the extrapolated curve.