Appendix A

Representation of Polarized Light

A.1 The Stoke's parameters

Several formalisms are available for representing radiation, but among those, the only one which can handle both polarized and unpolarized light simultaneously (or partially polarized light) is the Stoke's parameter representation which was developed by Sir George Stokes in the 1850s. In fact, this was the first mathematical formulation of partially polarized light. Unfortunately it was forgotten for about a century by physicists, until it was rediscovered and introduced into astronomy by S. Chandrasekhar. In the formalism of Stokes, light is represented in terms of four quantities all of which have the dimension of intensity and hence are measurable or are directly derivable from measured intensities. Below given are the relations between these quantities and the electric field components of radiation. But only those ideas and expressions which are directly relevant in the rest of this thesis, have been included here. More complete discussions are available in many excellent books such as Shurcliff (1962), Azzam & Bashara (1989), Collett (1993), Tinbergen (1996) etc. and the reviews referred to in the main text.

Quasi-monochromatic light can be represented in terms of two orthogonal linear polarization components, using the complex notation as

$$E_x = E_{x_0}(t) e^{i(\Phi + \phi_x(t))} \qquad \text{and} \qquad E_y = E_{y_0}(t) e^{i(\Phi + \phi_y(t))} \,, \tag{A.1}$$

where the propagation, $\Phi = \omega t + kz$. The four Stokes parameters are defined in terms of the electric fields as

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} < E_x E_x^\star > + < E_y E_y^\star > \\ < E_x E_x^\star > - < E_y E_y^\star > \\ < E_x E_y^\star > + < E_y E_x^\star > \\ i(< E_x E_y^\star > - < E_y E_x^\star >) \end{bmatrix} \,, \tag{A.2}$$

where the angular brackets $< >$ represent averaging over time scales in which the measurements are made. These time scales are usually much longer than that corresponding to $\frac{1}{\omega}$. The first Stokes parameter I is the total intensity of the light, the second one Q is the difference in intensities between two orthogonal linear polarization components measured in the reference coordinate system. The third parameter U is similar to Q, but measured in a coordinate system rotated by 45° with respect to the first one. Q and U are related to the fractional linear polarization p and its position angle θ by the relations,

$$Q = Ip\cos(2\theta) \qquad \text{and} \qquad U = Ip\sin(2\theta) \ . \tag{A.3}$$

V is the difference in intensities between the right and left circularly polarized components in the light and is called the circularity parameter. Substituting from Eq. A.1 reduces Eq. A.2 to

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} < E_{x_0}^2 + E_{y_0}^2 > \\ < E_{x_0}^2 - E_{y_0}^2 > \\ < 2E_{x_0}E_{y_0}\cos(\phi_x - \phi_y) > \\ < 2E_{x_0}E_{y_0}\sin(\phi_x - \phi_y) > \end{bmatrix} . \tag{A.4}$$

In general, for a given intensity I, the remaining three Stokes parameters can take any set of values which are restricted only by the inequality

$$I^2 \leq Q^2 + U^2 + V^2 \ . \tag{A.5}$$

Though the definitions in Eq. A.2 and Eq. A.4 were given for quasi-monochromatic light, they represent completely the polarization state of any light, independent of its bandwidth. As shown explicitly in Eq. A.1, the quantities E_{x_0}, ϕ_x, E_{y_0} and ϕ_y, in general, vary with time. The coherences that exist between the amplitudes E_{x_0} and E_{y_0} and/or the phases ϕ_x and ϕ_y, define the state of polarization of the light. For ideal, monochromatic light the amplitudes and phases are independent of time and in such a case, there is complete correlation between these quantities. The light is, then necessarily fully polarized and the constraint given by Eq. A.5 reduces to the equality. However, in practice, one can talk of only quasi-monochromatic light, in which the amplitudes and phases change slowly with respect to time scales corresponding to $\frac{1}{\omega}$. Quasi-monochromatic light may either be fully polarized or partially polarized depending on the correlation between the amplitudes E_{x_0} and E_{y_0} and/or between the phases ϕ_x and ϕ_y. In polychromatic light or white light, the magnitude of the electric field vector changes rapidly compared to $\frac{1}{\omega}$, in a non-periodic manner. Such light can be either fully polarized, partially polarized or unpolarized.

Since all the four Stokes parameters have dimensions of intensity, it is customary to divide them with the total intensity I. These normalized parameters are represented by 1, q, u, v.

Very often it becomes quite useful to represent the radiated electric field in terms of the acceleration components of the radiating charges in a polar coordinate system as shown below.

$$E_\theta = \left(\frac{q}{c^2 R}\right)[\ddot{x}\cos\theta - \ddot{z}\sin\theta] \qquad \text{and} \qquad E_\phi = \left(\frac{q}{c^2 R}\right)\ddot{y} \tag{A.6}$$

The Stoke's vector is then defined in terms of these components as follows.

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} < E_\phi E_\phi^\star > + < E_\theta E_\theta^\star > \\ < E_\phi E_\phi^\star > - < E_\theta E_\theta^\star > \\ < E_\phi E_\theta^\star > + < E_\theta E_\phi^\star > \\ i(< E_\phi E_\theta^\star > - < E_\theta E_\phi^\star >) \end{bmatrix} \tag{A.7}$$